AD/A-003 009

UNSTEADY HYDRODYNAMICS OF A BODY OF REVOLUTION WITH FAIRWATER AND RUDDER

Douglas P. Glasson

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Prepared for:

Naval Ship Research and Development Center

March 1974

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) UNSTEADY HYDRODYNAMICS OF A BODY OF REVOLUTION WITH FAIRWATER AND RUDDER		5. TYPE OF REPORT & PERIOD COVERED Final Report Oct. 71-March 74
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Douglas P. Glasson		8. CONTRACT OR GRANT NUMBER(s) N00014-67-A-0204-0058
9. PERFORMING ORGANIZATION NAME AND ADDRESS Massachusetts Institute of Technology Department of Ocean Engineering Cambridge, Mass. 02139		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE March 74
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; its distribution is unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

UNSTEADY LIFTING SURFACE
SUBMARINE CONTROL
SLENDER BODY

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Potential flow models are developed for a submerged body of revolution with fin and rudder appendages.

Forces and moments on the lifting surfaces and hull have been predicted at a steady angle of attack. The procedure is extended to the time dependent angle of attack case. Experimental, analytical and numerical approaches are described.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Ocean Engineering

Report 74-7

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This research was carried out under the Naval Ship Systems Command General Hydromechanics Research Program Subproject SR 009 01 01, administered by the Naval Ship Research and Development Center. Prepared under O.N.R. contract N00014-67-A-0204-0058. M.I.T. DSR 73836

Abstract

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Forces and moments on the lifting surfaces and hull have been predicted at a steady angle of attack. The procedure is extended to the time dependent angle of attack case. Experimental, analytical and numerical approaches are described.

ACKNOWLEDGEMENTS

The author extends his deepest appreciation to Dr. Damon Cummings for initially arranging for me to work on this project, and for his friendship and technical guidance throughout the past eight months.

The author is also deeply indebted to Nina Robinson and Barbara Olson for a blazing show of speed in typing this paper.

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NOMENCLATURE

AR	-	Aspect Ratio - (equivalent span)^2 /(plan area)
$C_n(x)$	-	n^{th} loading mode chordwise
$C_{mn}(s)$	-	Cross mode strength; m^{th} mode chordwise, n^{th} spanwise
C_L	-	Lift coefficient = (lift) / (1/2 ρU_∞^2 plan area)
H	-	Step size (fractions of sail chord)
L'	-	Roll Moment
N	-	Yaw Moment
P	-	Local pressure
r_0	-	Tip vortex radius
s	-	Distance traveled after sudden change in sideslip angle
t	-	Time elapsed after sudden change in sideslip angle
U_∞	-	Free Stream Velocity
u	-	Local perturbation velocity in x direction
$V_{x,y,z}$	-	Disturbance velocities due to wake in x, y, z directions
w	-	z Component of disturbance velocity due to sail and wake
Y	-	Side Force
α	-	Angle of attack
β	-	Sideslip angle
$\gamma(x)$	-	Local two dimensional vorticity
γ_i	-	Mode strengths for starting problem; i^{th} mode spanwise
γ_{in}	-	Mode strengths for steady problem; i^{th} mode chordwise; n^{th} spanwise

γ_{te_I} - Trailing edge value of sectional vorticity at I th time step

Γ_{2D_I} - Sectional bound vorticity at Ith time step

$\Delta\Phi$ - Local potential jump across wing-wake singularity distribution

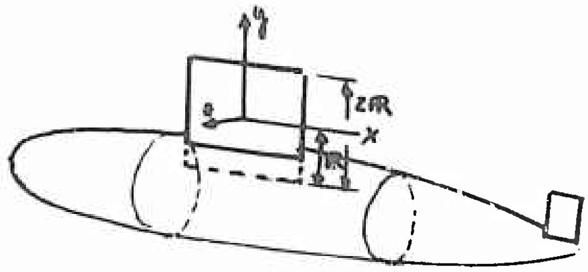
Φ - Velocity Potential

ρ - Fluid Density

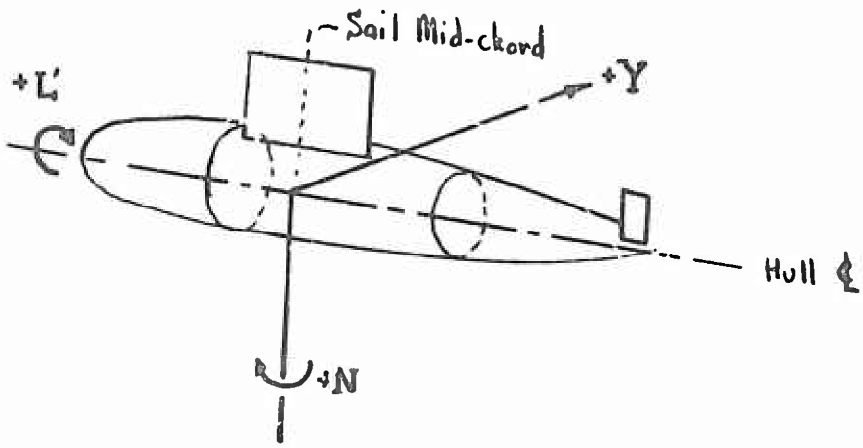
Θ - Wake Angle

Coordinate Axes

Hydrodynamic:



Forces & Moments:



Introduction

The primary aim of the work presented in this report is to add insight into dynamic interaction effects experienced by high speed submarines. The dramatic advances in submarine design since the second World War have been in the direction of higher speed, larger size and endurance, but depth capability, expressed in boat lengths or seconds has not increased proportionally. As a result, submarine control design, particularly for vertical plane motions and coupling between maneuvering control in the horizontal plane and response in the vertical plane has become of great importance. Its importance will undoubtably increase as submarine technology develops.

As a result of the demands on submarine control capability, understanding of the hydrodynamic forces acting on the boat has assumed greater importance and effects which were considered negligible or merely aggravating at low speed may be crucial at high speed.

The usual approach to solving problems in marine vehicle dynamics is to express the forces acting on the vehicle due to its motion as functions of its velocity and time derivatives of velocity. Such an approach is extremely useful since it lends itself conveniently to linearization about discrete operating points of interest and usually higher order derivative terms are of diminishing importance.

There are, however, disadvantages to this general approach. When the vehicle alters its environment in such a manner that the forces and moments acting upon it are dependent on its motion history over a significant time span, higher time derivatives become more important and the coefficients of these time derivatives become difficult to predict by theoretical or experimental techniques.

An example of such an effect on the vehicle environment is the trailing vortex system of a submarine sail. The relative location and strength of this vortex system depends on the circulation history of the sail, the

motion of the vortex sheet, and the trajectory and orientation of the vehicle. The vortex sheet has an effect on the hull and rudder forces which is strongly dependent on the relative location of the trailing sheet. Therefore the forces on the vehicle are dependent not only on its present velocity and acceleration, but on its motion history. To express this effect within the traditional framework demands enough higher time derivatives of the motion to recreate the vehicle history between the time the circulation formed on the sail to when it ceases to effect the rudder. There is of course no guarantee that higher derivatives will be of monotonically diminishing importance.

The approach taken in this work is to consider the hydrodynamic forces acting on the vehicle directly as a function of its motion history. The interactions between vortex sheets, hull, and rudder are derived directly from the geometry. This implies that for simulation work the motion history of vehicle and vortex sheets must be stored as the alternative to employing a large number of time derivatives.

The thrust of this effort is to develop techniques for evaluating hydrodynamic restoring forces in a history dependent situation. Various techniques have been employed, but the most promising for further pursuit seems to be a slender body approximation for the hull with singularity distribution models for the lifting surfaces. In both the slender body and lifting surface calculations, rough approximations and widely spaced grids are used.

The purpose of the authors was to develop techniques and evaluate the importance of effects not normally taken into account. Therefore little effort has been invested in refining numerical results. The difference between results including and not including interactions is the point of interest. If the techniques investigated are to be used for simulation or prediction of submarine motions, the numerical procedures described require considerable refinement.

The total effort divides into several logical parts. First an experimental study was conducted in the MIT variable pressure water tunnel of a small submarine model with various fin configurations. Forces and moments were measured at various angles of attack. Of greatest importance in the experimental study, however, were the visual results. The facility used has the great advantage of plexiglass walls and control of ambient pressure. By lowering the tunnel pressure, cavitation bubbles can be induced in the trailing vortex system from the sail so that its trajectory may be observed. In addition tufts may be attached to the hull and fins to observe flow patterns around the model.

These studies are reported by Luckard (1) under separate cover.

An analytical approach to the interaction problem was pursued by Newman and Rodriugez making slender body and low-aspect ratio assumptions and linearizing the problem. This approach is described by Newman and Wu (2).

The bulk of the work was devoted to numerical approaches to representation of the hydrodynamics problems. The approach assumes that the hull is slender and a body of revolution but includes finite aspect ratio lifting surfaces. The boundary value problems are approximately solved with discrete singularity distribution representations of hull, lifting surfaces and rotational wake.

Experimental Approach

The single greatest question at the initiation of this work was where the trailing vortex wake from the sail went. To obtain linear approximations to the hull forces as a function of sideslip angle it is necessary to assume that the vortex sheet trajectory is not dependent on angle of attack. For small sideslip angles the trailing vortex sheet from the sail is assumed to remain in the plane of the rudder axis and longitudinal

centerline. In this location the sail wake has maximum effect on rudder lift. If in practice the sail wake does not pass close to the rudder, this effect will be appreciably reduced. The path of this trailing vortex sheet is influenced by its own induced velocities as well as the free stream and the vehicle boundaries. Approximations may be made on the basis of lifting line theory and singularity distributions for the trajectory, but the tendency of the sheet to roll up makes such computation questionable.

For this reason photography of actual vortex sheet paths at steady angles of attack was valuable input to the modeling.

The model used for the experimental and numerical work is two foot long 'submarine like' body of revolution equipped with removable fairwater and upper and lower fins for testing in the MIT variable pressure water tunnel. Perhaps the most important result of this experimental and analytical effort is that, at small angles of attack, the lift on the rudder behind the sail is reversed from that predicted if the wake of the fairwater is ignored. This is of importance primarily to directional stability predictions of submarine type hulls. The effect of the rudder is destabilizing rather than stabilizing due to its interaction with the fairwater wake. The most recent work on the unsteady hydrodynamics problem comprises the technical content of this report since the experimental and steady numerical approaches are included in the report by Luckard (1). Comparisons of results for the steady case with the Newman and Wu (2) analytical results are also presented by Luckard (1).

The part of this project that is probably of most general interest is the approach taken to numerical solutions of the forces on fairwater, hull, and rudder due to a sudden change in angle of attack.

II. The Response of a Submarine Sail to a Sudden Change in Slideslip Angle

A. Unsteady Lifting Surface Theory

The transient buildup of lift and moment on an initially unloaded lifting surface that has undergone a sudden change in angle of attack is a direct result of the physical fact that a finite time is required for the trailing vortex sheet of the lifting surface to attain its steady state configuration. At the instant that the surface's orientation to the flow is changed, all vorticity is confined to the surface itself. As the surface moves forward in its new orientation, vorticity is shed from the surface into the external flow. Eventually, the lifting surface reaches a steady state condition in which the wake has attained its familiar "trailing vortex sheet" structure, except for regions of the wake that can be considered to be an infinite distance from the lifting surface.

Using a distribution of dipoles to represent the lifting surface and its wake at a given instant of time, the z component of velocity is given by the following expression: (Derivation is in Appendix A)

$$(II.1) \qquad w(x_0,y_0,z_0,t) = -\frac{1}{4\pi} \oint\!\!\oint_{\substack{\text{Sail}\\+\\\text{Wake}}} \frac{\partial^2 \tau(x,y,t)}{\partial x\, \partial y} \left\{ \frac{1}{(x-x_0)^2+z_0^2} + \frac{1}{(y-y_0)^2+z_0^2} \right\} \frac{(x-x_0)(y-y_0)\, dx\, dy}{\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}}$$

where τ is the local dipole sheet strength, its second mixed derivative expressing the local strength of the related point-horseshoe vortex sheet. Applying the boundary condition that there is no flow normal to the lifting surface boundary at $z=0$ one obtains:

$$-w(x_0, y_0, 0^+, t) = U_\infty(\alpha)$$

$$= \frac{1}{4\pi} \iint_{\text{Sail}} \frac{\partial^2 \Gamma(x,y,t)}{\partial x \partial y} \frac{\sqrt{(x-x_0)^2 + (y-y_0)^2}}{(x-x_0)(y-y_0)} \, dx \, dy$$

$$+ \frac{1}{4\pi} \iint_{\text{Wake}} \frac{\partial^2 \Gamma(x,y,t)}{\partial x \partial y} \left\{ \frac{1}{(x-x_0)^2 + z_0^2} + \frac{1}{(y-y_0)^2 + z_0^2} \right\} \times \frac{(x-x_0)(y-y_0)}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + z_0^2}} \, dx \, dy$$

(II.2)

The integral over the wake has been written in this manner to allow the option of placing the wake in a plane other than $z = 0$. Equation (II.2) is the mathematical statement of the relationship between the known boundary conditions for the lifting surface and the unknown singularity distribution representing the lifting surface and its wake.

The irrotationality condition on the flow external to the lifting surface and its wake generates an important relationship between the distribution of vorticity in the wake and the history of the total vorticity bound to the lifting surface. Consider a two dimensional section of the wing-wake flow. In order for the value of $\Delta\Phi$ (the local dipole sheet strength) to be single valued at the trailing edge of the lifting surface:

$$\Delta\Phi_{TE} = \oint_{C_A} \vec{V} \cdot d\vec{l} = \oint_{C_B} \vec{V} \cdot d\vec{l}$$

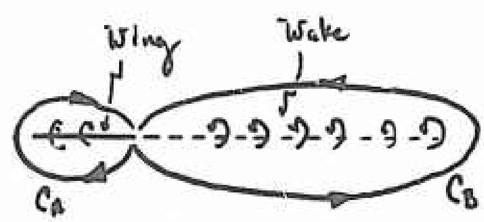


Figure II.1

Since $\oint_{C_A} \vec{V} \cdot d\vec{l}$ is the total vorticity strength enclosed by the C_A contour and, hence, the total bound vorticity on the wing at the moment of interest, this relationship states that the total amount of vorticity shed into the wake equals the negative of the present amount of vorticity bound to the lifting surface. Consider this process of bound vorticity increase and shedding of vorticity into the wake:

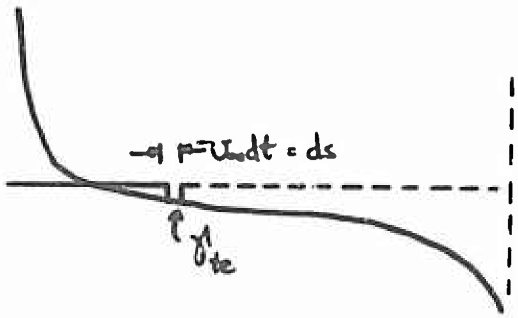


Figure (II.2)

If, at any section, the total amount of bound vorticity increases by $d\Gamma_{2D}$ in dt amount of time, then an amount of circulation equal to $\gamma_{te} U_\infty dt$ must have been shed into the wake such that:

$$d\Gamma_{2D} = -\gamma_{t.e.} U_\infty dt = -\gamma_{te}\, ds$$

$$\therefore \; \gamma_{te} = -\frac{d\Gamma_{2D}}{ds} \qquad (II.4)$$

Now

$$\left.\frac{\partial^2 \gamma(x,y,t)}{\partial x \partial y}\right|_{wake} = \frac{\partial}{\partial y}\left(\gamma_{te}(x-s,y)\right)$$

$$= \frac{\partial}{\partial y}\left(-\frac{d\Gamma_{2D}}{ds}(x-s,y)\right)$$

$$= \frac{\partial}{\partial s}\left(\frac{\partial \Gamma_{2D}}{\partial y}(x-s,y)\right) \qquad (II.5)$$

Substituting into (II.2), the boundary value integral equation becomes:

$$U_\infty \alpha(t) = \frac{1}{4\pi}\iint_{Wing} \frac{\partial^2 \gamma}{\partial x \partial y} \frac{\sqrt{(x-x_0)^2+(y-y_0)^2}}{(x-x_0)(y-y_0)}\, dx\, dy$$

$$+\frac{1}{4\pi}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_0^s \frac{\partial}{\partial s}\left(\frac{\partial \Gamma_{2D}}{\partial y}(x-s,y)\right)\left\{\frac{1}{(x-x_0)^2+z_0^2}+\frac{1}{(y-y_0)^2+z_0^2}\right\}\frac{(x-x_0)(y-y_0)\, dx\, dy}{\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}} \qquad (II.6)$$

Hence, the integral over the wake has been reduced to a convolution integral involving (known) previous values of the total bound circulation at each section of the lifting surface.

In order to calculate the forces and moments on the lifting surface as a function of distance traveled, it is necessary to consider the time-dependent form of the Bernoulli equation.

$$\rho \frac{\partial \Phi_1}{\partial t} + \frac{1}{2}\rho V_1^2 + p_1 + \rho g h_1 = \rho \frac{\partial \Phi_2}{\partial t} + \frac{1}{2}\rho V_2^2 + p_2 + \rho g h_2 \qquad \text{(II.7)}$$

where ① and ② denote points on a streamline

Applying this equation to points on the upper and lower surfaces of a lifting surface section with a steady, uniform free stream one obtains:

$$\rho \frac{\partial \Phi_u}{\partial t} + \frac{1}{2}\rho (U_\infty + u_u)^2 + p_u = \rho \frac{\partial \Phi_L}{\partial t} + \frac{1}{2}\rho (U_\infty + u_L)^2 + p_L$$

$$(p_L - p_u) = \Delta p = \rho \frac{\partial}{\partial t}(\Delta\Phi(x)) + \rho U_\infty \gamma(x) = \rho U_\infty \left\{ \frac{\partial}{\partial s}(\Delta\Phi(x)) + \gamma(x) \right\}$$

where $\gamma(x) = 2u_u = -2u_L$

This expression can also be written:

$$\Delta p = \rho U_\infty \left\{ \frac{\partial}{\partial s} \int_{-1}^{x} \gamma(\xi)\, d\xi + \gamma(x) \right\} \quad (\mathrm{II}.8)$$

Δp is then integrated in the usual manner to yield the total force and moment on the lifting surface.

Although a complete solution of the problem of the transient response of a lifting surface requires a solution of equation (II.6), some important trends can be deduced by considering the starting instant and steady state solutions of equation (II.6). For a flat, rectangular wing of unit half chord with a planar wake, equation (II.6) reduces to:

$$(\mathrm{II}.9) \quad \alpha = \frac{1}{4\pi U_\infty} \int_{-R}^{R} \int_{-1}^{1} \frac{\partial^2 \Gamma}{\partial x \partial y} \frac{\sqrt{(x-x_0)^2 + (y-y_0)^2}}{(x-x_0)(y-y_0)}\, dx\, dy \quad \text{(starting problem)}$$

$$(\mathrm{II}.10) \quad \alpha = \frac{1}{4\pi U_\infty} \int_{-R}^{R} \int_{-1}^{1} \frac{\partial^2 \Gamma}{\partial x \partial y} \frac{\sqrt{(x-x_0)^2 + (y-y_0)^2}}{(x-x_0)(y-y_0)}\, dx\, dy + \frac{1}{4\pi U_\infty} \int_{-R}^{R} \frac{\partial \Gamma(1,y)}{\partial y} \frac{dy}{y-y_0} \quad \text{(steady problem)}$$

It has been shown by Wagner (4) that in the two dimensional $(R = \infty)$ limits of the above equations the starting problem is satisfied by a vorticity mode of the functional form:

$$\left. \frac{\gamma(x)}{U_\infty} \right|_{\text{start}} = 2\alpha \frac{-x}{\sqrt{1-x^2}} \quad (\mathrm{II}.11)$$

The result for the steady two-dimensional case from classical thin airfoil theory is:

$$\left.\frac{\gamma(x)}{U_\infty}\right|_{steady} = 2\alpha\sqrt{\frac{1-x}{1+x}} \qquad (II.12)$$

The sectional lift for the starting problem is, Ref. (1):

$$C_{L_{start}} = \pi\alpha \qquad (II.13)$$

and for the steady problem:

$$C_{L_{steady}} = 2\pi\alpha \quad (II.14) \quad ; \quad \therefore \quad \frac{C_{L\,steady}}{C_{L\,start}} = 2.$$

For finite aspect ratio wings the presence of the trailing vortex wake and its associated downwash causes a decrease in the ratio of the final and initial lifts. As the aspect ratio of a finite wing is decreased, the wake makes an increasingly dominant contribution to the boundary condition equation (II.6), until in the limit of zero aspect ratio, the boundary condition is assumed to be satisfied exclusively by the trailing vortex sheet. It seems reasonable that for sufficiently low aspect ratio the starting lift may exceed the steady state lift due to the predominance of the trailing vortex sheet downwash in the steady case.

A numerical study was carried out to compare the starting and steady lifts of isolated rectangular wings of various aspect ratioes. The Integral equation (II.9) was solved for the starting problem assuming a loading expansion of the form:

$$\frac{\partial\Gamma}{\partial x} = \sum_{i=1}^{3} \gamma_i \frac{-x}{\sqrt{1-x^2}} S_i(y) \qquad (II.15)$$

i.e. the chordwise mode shape for the two-dimensional case multiplied by some spanwise distribution function. In the steady case, the loading expansion assumed was of the form:

$$\frac{\partial \Gamma}{\partial x} = \sum_{i=1}^{3} \sum_{n=1}^{2} \gamma_{in}\, C_n(x)\, S_i(y) \quad (II.6)$$

where:

$$C_n(x) = \begin{cases} \sqrt{\frac{1-x}{1+x}} \; ; & n=1 \\ \sqrt{\frac{1-x}{1+x}} - 2\sqrt{1-x^2} \; ; & n=2 \end{cases} \quad (II.7)$$

and the $S_i(y)$ are given by:

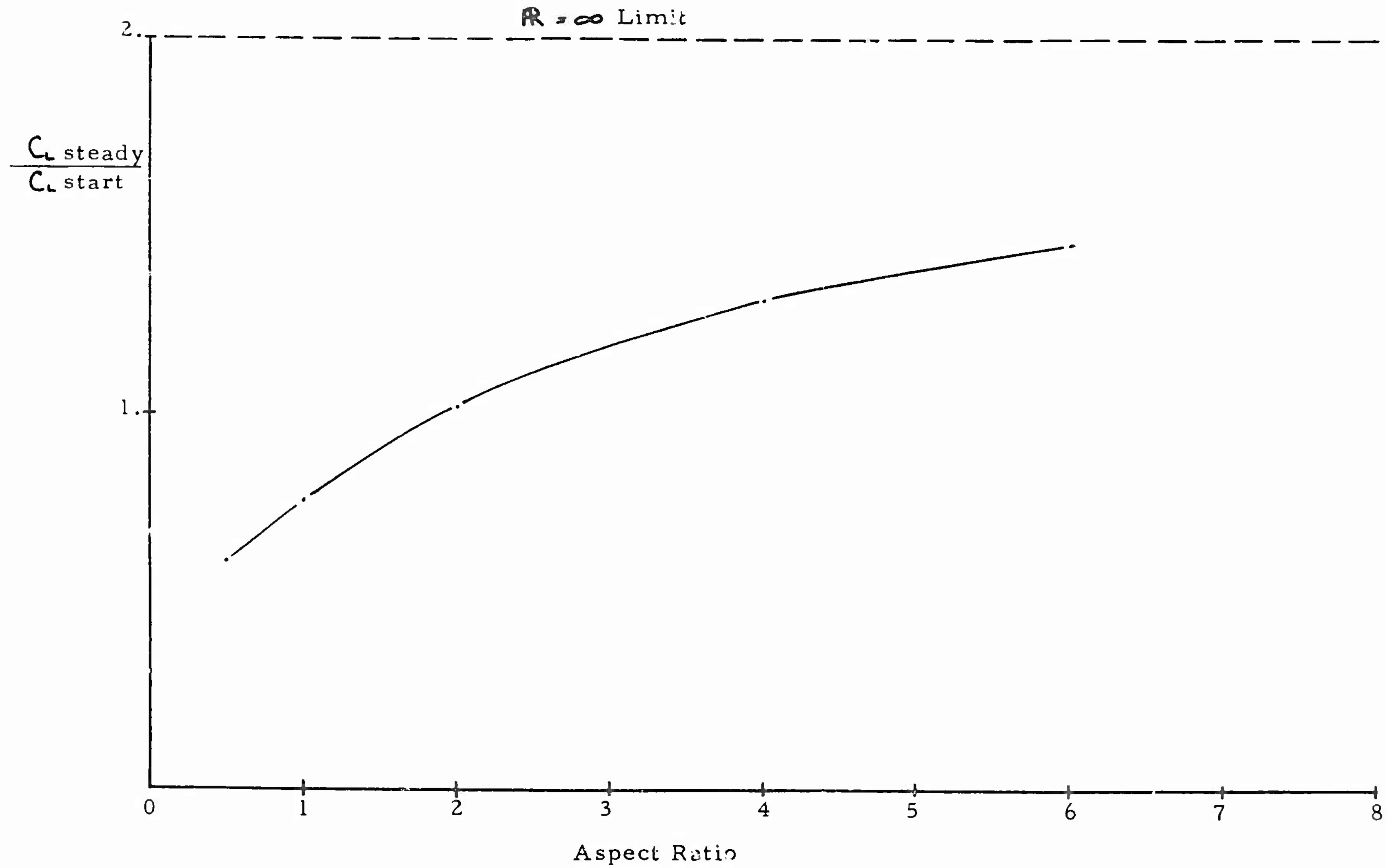
$$S_i(y) = \begin{cases} \sin\Theta \; ; & i=1 \\ \sin 3\Theta \; ; & i=2 \\ \sin 5\Theta \; ; & i=3 \end{cases} \qquad \Theta = \sin^{-1}\left(\frac{y}{R}\right)$$

In all cases, the unknown coefficients of the assumed loading modes were obtained by applying equation (II.6) at 16 evenly distributed control points and finding a best-least-squares fit of the γ_i's to the (over determined) set of linear equations that resulted. The starting and steady forces were then calculated by integrating the sectional forces given by equations (II.13) and (II.14) across the span.

The resulting values of C_L steady/C_L start are plotted below. The results show that the starting and steady lifts are approximately equal at an aspect ratio of 2. At an aspect ratio of 1. (a typical value for the aspect ratio of a modern submarine sail) the starting lift exceeds the final lift by roughly 25%. Hence, a significant quantitative difference between the forces calculated in the transient (step response) of a low ($\sim$1) aspect ratio wing and what would be predicted in a psuedo-steady analysis is already apparent.

Figure II. 3

Ratio of Final Lift to Starting Lift in Step Response versus Aspect Ratio, Rectangular Wings



B. Indicial Response of a Submarine Sail:

1) The Image System

Since the sail is attached to what often can be considered a locally cylindrical surface (the submarine hull), an image system is required to approximately satisfy the boundary condition of flow tangency to the hull. The theoretical basis for the image system used is given in Milne - Thompson (5) and is reviewed by Luckard (1) for the particular case of a hull-sail combination. Using Luckard's results for the calculation of the equivalent span of the sail, one obtains:

$$R_{eq} = \frac{1}{2}\,\frac{(RSL^2 - RHS^2)}{RSL} \qquad (II.18)$$

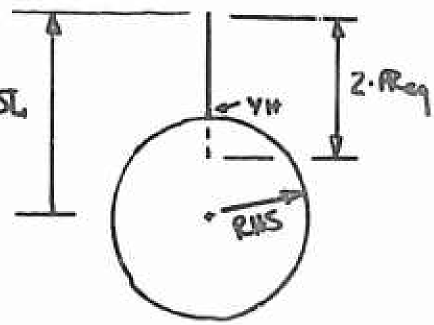


Figure (II.1)

where RSL and RHS are given in terms of half chords of the sail. In addition to determining the equivalent span, the presence of the hull causes an increase in the in-flow velocity to the sail given by (1):

$$\Delta\beta = U_\infty \beta \frac{r_0^2}{RHS^2}$$

Hence, the local angle of attack seen at a control point (x_0, y_0) on the sail is given by:

$$\beta_{local} = U_\infty \beta \left\{ 1 + \frac{r_0^2}{RHS^2} \right\}$$

$$= U_\infty \beta \left\{ 1 + \left(\frac{RSL - R + y_0}{RHS} \right)^2 \right\}$$

2) The Spanwise Loading Modes:

The tangency condition (on the hull surface) also requires that the sheet of trailing vorticity have zero strength at the hull surface, i.e. the discontinuity in the y perturbation velocity across the trailing vortex sheet must vanish at the hull-sail junction. This requires that the slopes of the spanwise loading functions assumed to solve equation (II.6) be zero at the hull-sail junction. To satisfy this requirement, spanwise loading modes of the forms given below were chosen:

$$
\begin{aligned}
S_1(y) &= \sin\left\{\frac{y'+1}{YH+1}\cdot\frac{\pi}{2}\right\}; \quad -1 \le y' \le YH \\
&= \sin\left\{\frac{y'-1}{YH-1}\cdot\frac{\pi}{2}\right\}; \quad YH \le y' \le 1 \\
S_2(y) &= \sin\left\{\frac{y'+1}{YH+1}\cdot\frac{\pi}{2}\right\}; \quad -1 \le y' \le YH \\
&= \sin\left\{\frac{y'}{YH}\cdot\frac{\pi}{2}\right\}; \quad YH \le y' \le 0 \\
&= -\sin\left\{y'\pi\right\}; \quad 0 \le y \le 1 \\
S_3(y) &= \sin\left\{\frac{y'+1}{YH+1}\cdot\frac{3\pi}{2}\right\}; \quad -1 \le y' \le YH \\
&= \sin\left\{\frac{y'-1}{YH-1}\cdot\frac{3\pi}{2}\right\}; \quad YH \le y' \le 1
\end{aligned}
\qquad (II.19)
$$

where: $y = R\sin\left(\frac{\pi}{2}y'\right)$; or $y' = \frac{2}{\pi}\sin^{-1}\left(\frac{y}{R}\right)$

3) The Chordwise Loading Modes

In addition to the chordwise loading modes given by (II.17) a mode is required that can provide for a non-zero value of vorticity at the trailing edge of the sail. Since the starting mode given by (II.11) yields

a nearl constant chordwise downwash for the aspect ratio range of interest, a special mode was constructed having the following functional form:

$$C_0(x) = \frac{\left(\frac{x - H/2}{1 + H/2}\right)\frac{1}{1 + H/2}}{\sqrt{1 - \left(\frac{x - H/2}{1 + H/2}\right)^2}} - \frac{\sqrt{1 - \left(\frac{1 - H/2}{1 + H/2}\right)^2}}{\pi}\sqrt{\frac{1 - x}{1 + x}} \qquad \text{(II.20)}$$

$$\frac{H}{2} = .0625 \text{ (typically)}$$

The first term represents the chordwise vorticity distribution given by (II.11) where the singular trailing edge portion of the distribution has been moved past the trailing edge of the chord by replacing X with $\left(\frac{x - H/2}{1 + H/2}\right)$ The second term is subtracted in order to make the net circulation of the special mode equal to zero.

Since equation (II.4) relates the trailing edge value of vorticity to the total net vorticity bound to the lifting surface, it is convenient to normalize the trailing edge value of vorticity for this chordwise mode by dividing through by $C_0(1)$

$$C_0'(x) = \left(\frac{x - \frac{H}{2}}{1 - H/2}\right)\sqrt{\frac{1 - \left(\frac{1 - H/2}{1 + H/2}\right)^2}{1 - \left(\frac{x - H/2}{1 + H/2}\right)^2}} - \frac{2H}{\pi(1 - H/2)}\sqrt{\frac{1 - x}{1 + x}} \qquad \text{(II.21)}$$

4) The Wake

The distribution of vorticity in the wake at any point in time was assumed to have a y variation that could be described by the spanwise modes (II.19) and a piecewise linear x variation. The piecewise linear variation in x was chosen since this reduces the convolution integral representing the downwash of the wake in equation (II.6) to a simple matrix multiplication of a downwash influence matrix by the previous values of the total bound vorticity on the lifting surface. A piecewise linear distribution also has the advantage that the x integration of the second term of equation (II.6) can be carried out analytically.

The trailing edge of the wake for the indicial lift case has a square-root singularity of vorticity strength in x. Since this distribution can be comparatively troublesome to integrate numerically or otherwise, this portion of the wake was modeled by a piecewise linear portion plus an impulse of vorticity at the trailing edge. The relative values of the piecewise linear portion and the impulse were adjusted to make the areas and centroids of thetwo distributions equal.

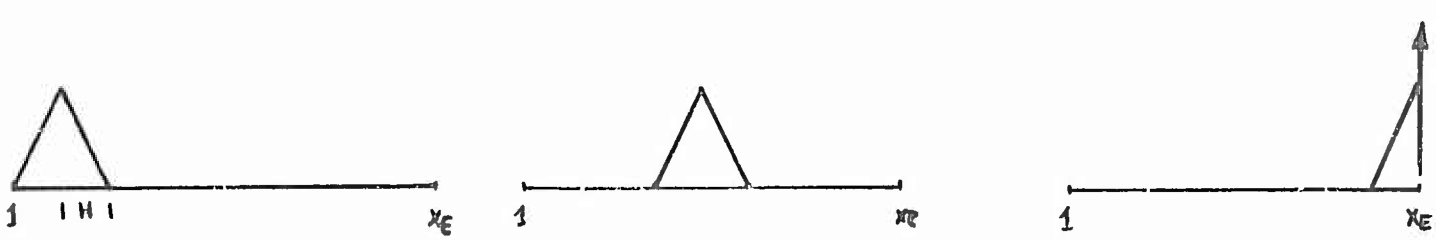


Figure (II.5) - Wake Vorticity Modes

5) Step by Step Solution:

At this point, equation (II.6) can be written as:

$$\beta(x_0,y_0,s) = \sum_n \sum_m C_{mn}(s)\, DWS_{mn}(x_0,y_0) + \sum_n \int_0^s DWI_n(x_0,y_0,\tau)\, \frac{\partial \Gamma_{2D}(s-\tau)}{\partial s}\, d\tau$$

$$+ \sum_n \frac{\partial \Gamma_{2D_n}(s)}{\partial s}\, DWS_{0n}(x_0,y_0) \qquad \text{(II.22)}$$

where β is the local sideslip angle; $DWS_{mn}(x_0,y_0)$ is the sidewash at control point (x_0, y_0) on the sail due to a vorticity cross mode m chordwise - n spanwise. $C_{mn}(s)$ is the coefficient of crossmode mn. $DWI_n(x_0,y_0,\tau)$ is the sidewash at control point (x_0, y_0) on the sail due to a spanwise strip of the wake vorticity distribution located a distance τ from the trailing edge of the sail. $DWS_{0n}(x_0,y_0)$ is the sidewash due to the special chordwise mode given by (II.21). This equation is to be solved for a discrete set of distances traveled s.

Since the distribution of vorticity in the wake of the sail was assumed to be piecewise linear, relationship (II.4) must be finitized. Consider the balance of wake and bound vorticity at a section of the sail during a finite interval of time $\frac{H}{U_\infty}$ with the assumption that the vorticity of the wake is linear between x = l and x = l + H. Relationship (II.3) requires that:

$$\Gamma_{2D\,I} - \Gamma_{2D\,I-1} = -\frac{H}{2}\left(\gamma_{te_I} + \gamma_{te\,I-1}\right)$$

$$\gamma_{te_I} = -\frac{\Gamma_{2D\,I} - \Gamma_{2D\,I-1}}{\frac{H}{2}} - \gamma_{te\,I-1} \qquad \text{(II.23)}$$

With this result, (II.22) is rewritten:

$$\beta(x_0,y_0,N\cdot H) = \sum_n \sum_m C_{mn}(N\cdot H)\cdot DWS_{mn}(x_0,y_0) + \sum_n \sum_{I=2}^{N-1} DWT_n(x_0,y_0,(N-I)\cdot H)\cdot \gamma_{te_n I}$$

$$+ \sum_n -\left\{ \frac{\Gamma_{20_n N} - \Gamma_{20_n N-1}}{\frac{H}{2}} + \gamma_{te_n N-1} \right\} DWS_{on}(x_0,y_0) + \sum_n \gamma_{te_n 1} DWTE_n(x_0,y_0,N\cdot H)$$

$$\text{(II.24)}$$

where $DWTE_n$ is the sidewash due to the wake trailing edge mode described in section (II.A.4), with spanwise variation n; and $DWT_n(x_0,y_0,N\cdot H)$ is the sidewash due to a triangular distribution of vorticity (figure II.5) having its vertex located a distance $N\cdot H$ from the trailing edge of the sail with spanwise variation n.

Moving all known quantities to the right hand side of the equation:

$$\sum_n \sum_m C_{mn}(N\cdot H)\cdot DWS_{mn}(x_0,y_0) - \sum_n DWS_{on}(x_0,y_0)\cdot\left(\frac{\Gamma_{20_n N}}{\frac{H}{2}}\right)$$

$$= \beta(x_0,y_0,N\cdot H) + \sum_n \sum_{I=2}^{N-1} DWT_n(x_0,y_0,(N-I)\cdot H)\cdot \gamma_{te_n I}$$

$$+ \sum_n \left(\frac{\Gamma_{20_n N-1}}{\frac{H}{2}} + \gamma_{te_n N-1} \right\} DWS_{on}(x_0,y_0) + \sum_n \gamma_{te_n 1} DWTE(x_0,y_0,N\cdot H)$$

$$\text{(II.25)}$$

Of the chordwise modes chosen, only the first mode chordwise makes any contribution to Γ_{20_n}:

Therefore, the final form of (II.6) is:

$$\sum_n \sum_m C_{mn}(N\cdot H)\cdot DWS_{mn}(x_0,y_0) - \frac{2\pi}{H}\sum_n C_{1n}(N\cdot H)\cdot DWS_{on}(x_0,y_0)$$

$$= \beta(x_0,y_0,N\cdot H) - \sum_n \sum_{I=2}^{N-1} DWT_n(x_0,y_0,(N-I)\cdot H)\cdot \gamma_{te_n I} + \sum_n \left\{ \frac{2\pi}{H} C_{1n_{N-1}} + \gamma_{te_{n_{N-1}}} \right\} DWS_{on}(x_0,y_0)$$

$$- \sum_n \gamma_{te_n 1} DWTE_n(x_0,y_0,N\cdot H) \qquad \text{(II.27)}$$

In this form, the solution procedure is reduced to the following:

1) Choose a set of control points,
2) Construct (by numerical integration) the left hand-side of (II.27) in matrix form.
3) Construct (by numerical integration) a table of $DWT_n(x_0, y_0, N\cdot H)$ and $DWTE_n(x_0, y_0, N\cdot H)$ for each spanwise mode at each control point for a sequence of steps downstream.
4) Construct the right-hand side of (II.27) from previously calculated values of γ_{ten_J}, the results of step 3, and numerical integrations to determine $DJSon(x_0, y_0)$ at the control points.
5) Solve the resulting (over-determined) system of equations by the method of least squares for the unknown values of $C_{mn}(N\cdot H)$.
6) Calculate the new values of γ_{ten_N} by equation (II.23).
7) Increase N by 1 and return to step 4.

Note that for the first time step, γ_{ten_1} becomes an unknown, and the equation (II.27) takes the form:

$$\sum_n \sum_m C_{mn}(N\cdot H)\cdot DWS_{mn}(x_0, y_0) + \sum_n \gamma_{ten_1}\left\{DWS_{on}(x_0, y_0) + DWTE_n(x_0, y_0, H)\right\}$$
$$= \beta(x_0, y_0, H)$$

6) Force and Moment Calculations

The sectional force and moment produced by each chordwise mode is obtained by integrating (II. 8) across the chord of the sail section. Identifying the "unsteady" part of Δp as the $\rho U_\infty \frac{\partial}{\partial s}\int_0^x \gamma(\xi)d\xi$ term and the "steady" part as the $\rho U_\infty \gamma(x)$ term one obtains:

Steady Terms:

Mode Shape	Sectional Force	Sectional Moment
$\gamma(x)$	$= \int_{-1}^{1} \rho U_\infty \gamma(x)\,dx$	$= -\int_{-1}^{1} \rho U_\infty \gamma(x)\, x\,dx$
$U_\infty \sqrt{\frac{1-x}{1+x}}$	$\rho U_\infty^2 \pi$	$\frac{1}{2}\rho U_\infty^2 \pi$
$U_\infty \left\{ \sqrt{\frac{1-x}{1+x}} - 2\sqrt{1-x^2} \right\}$	0	$\frac{1}{4}\rho U_\infty^2 \pi$
$U_\infty \left\{ \left(\frac{x-\frac{H}{2}}{1+\frac{H}{2}}\right) \sqrt{\frac{1-\left(\frac{1-\frac{H}{2}}{1+H/2}\right)^2}{1-\left(\frac{x-H/2}{1+H/2}\right)^2}} - \frac{2H}{\pi(1-H/2)}\sqrt{\frac{1-x}{1+x}} \right\}$	0	$\frac{\rho U_\infty^2}{1-H/2}\left\{ H + \sqrt{\frac{H}{2}}\left[\frac{\pi}{2} + \frac{(1-H/2)}{(1+\frac{H}{2})^2} 2\sqrt{\frac{H}{2}} + \sin^{-1}\left(\frac{1-\frac{H}{2}}{1+H/2}\right)\right]\right\}$

Unsteady Terms:

Mode Shape	Sectional Force	Sectional Moment
$\gamma(x)$	$= \int_{-1}^{1} \rho U_\infty \int_{-1}^{x} \gamma(\xi)\,d\xi\,dx$	$= \int_{-1}^{1} \rho U_\infty x \int_{-1}^{x} \gamma(\xi)\,d\xi\,dx$
$U_\infty \sqrt{\frac{1-x}{1+x}}$	$\frac{3}{2}\rho U_\infty^2 \pi$	$-\frac{1}{4}\rho U_\infty^2 \pi$
$U_\infty \left\{ \sqrt{\frac{1-x}{1+x}} - 2\sqrt{1-x^2} \right\}$	$\frac{1}{4}\rho U_\infty^2 \pi$	$\frac{1}{8}\rho U_\infty^2 \pi$

$$U_\infty \left\{ \left(\frac{x-\frac{H}{2}}{1+\frac{H}{2}}\right) \sqrt{\frac{1-\left(\frac{1-\frac{H}{2}}{1+\frac{H}{2}}\right)^2}{1-\left(\frac{x-\frac{H}{2}}{1+\frac{H}{2}}\right)^2}} - \frac{2H}{\pi\left(1+\frac{H}{2}\right)}\sqrt{\frac{1-x}{1+x}} \right\} \quad \left\{ \frac{\sqrt{\frac{H}{2}}}{1-\frac{H}{2}} \left\{ \frac{\pi}{2} + \frac{1-\frac{H}{2}}{\left(1+\frac{H}{2}\right)^2} 2\sqrt{\frac{H}{2}} + \sin^{-1}\left(\frac{1-\frac{H}{2}}{1+\frac{H}{2}}\right) \right\} - \frac{3H}{1-\frac{H}{2}} \right\} \cdot \rho U_\infty^2 \quad \left\{ -\frac{H}{2}\left(1+\frac{H}{2}\right)^2 \cdot S.F. + \frac{\frac{1}{3}H^2}{1-\frac{H}{2}} + \frac{H/2}{1-\frac{H}{2}} \right\} \cdot \rho U_\infty^2$$

where: S. F. denotes the unsteady sectional force for the same mode.

Note that the total sectional force for each mode is given by the local sectional mode strength times the steady term plus the rate of change of the local sectional mode strength with distance traveled times the unsteady term. These sectional forces and moments are then integrated (numerically) across the span of the sail to yield the total side force, yaw moment, and roll moment on the lifting surface.

III. Hull-Wake Interaction

A. Wake Trajectory

As a result of Kelvin's vortex theorem, the trailing vortices of the sail wake must follow the streamlines of the flow about the hull. The trailing vortex emanating from the tip of the sail, for example, must follow the streamline trajectory shown below.

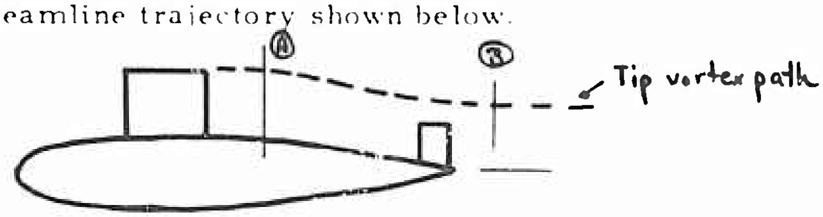
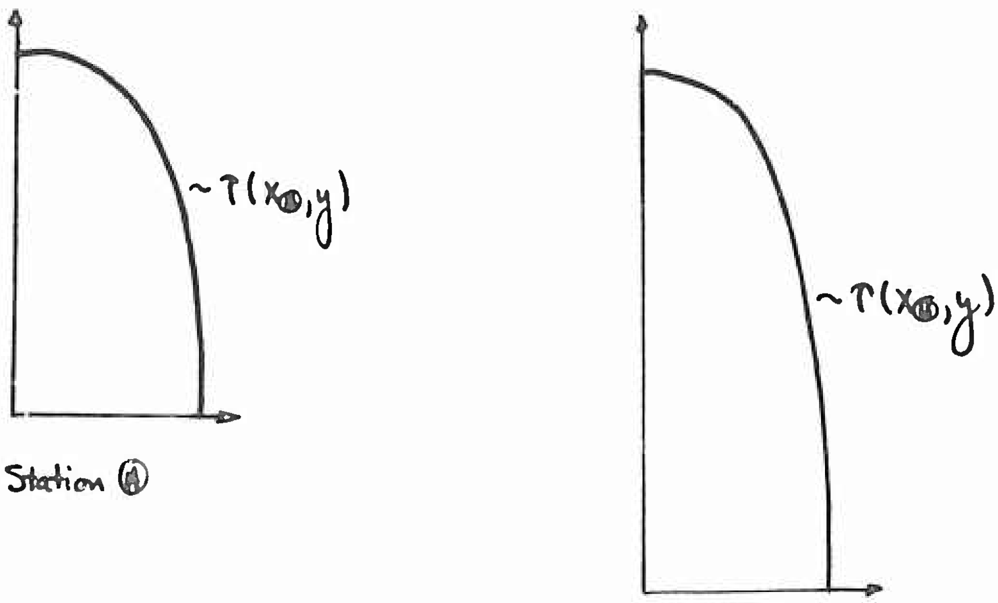


Figure (III.1)

As a rough approximation to the so-called wake contraction trajectory, the wake was assumed to situate itself between the sail tip streamline and the hull surface; at any station the vorticity distribution was assumed to have stretched linearly.



Station Ⓐ

Station Ⓑ

Figure(III.2)
Vorticity Distribution Stretching

The sail tip streamline trajectory is calculated in a step by step manner from the following relationship:

$$\frac{r_{on} - r_{on-1}}{x_{on} - x_{on-1}} = \frac{-r_{o\,n-1}}{2\pi} \int_{\text{Bow}}^{\text{Stern}} \frac{dS(x)}{dx} \frac{dx}{\left\{(x - x_{on-1})^2 + r_{o\,n-1}^2\right\}^{3/2}} \qquad \text{(II.1)}$$

which states that the local streamline slope is approximately equal to the local radial velocity produced by the source distribution representing the hull, divided by the free stream velocity.

B. The Interaction Response of the Hull to a Step Change in Sail Circulation

The simplest means of analyzing the unsteady forces on the hull caused by the velocity field of the sail wake following a sudden change in sideslip angle is to consider the response of the hull to a step change in sail circulation, then use the superposition theorem (convolution integral). In essence, the wake produced during the indicial response of the sail is considered to be constructed from the wakes due to an infinity of infinitesimal step changes in circulation.

The distribution of vorticity in the sail wake following a step change in sail circulation is as shown below:

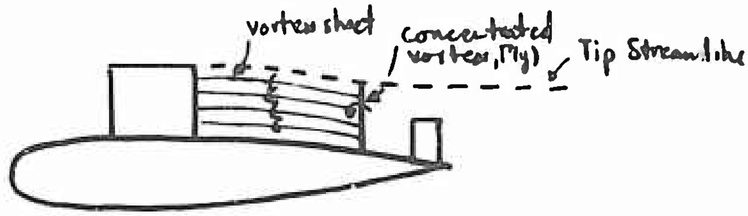


Figure (III.3)

The wake resembles a lifting line moving along the wake trajectory, its trailing vortices lengthening at a rate equal to the free stream velocity. For simplicity of calculation, the trailing vortices were considered to be piecewise straight as shown in Figure III. 4. This simplification should introduce little error since submarine hulls are typically very slender bodies (having uniformly small slope and curvature).

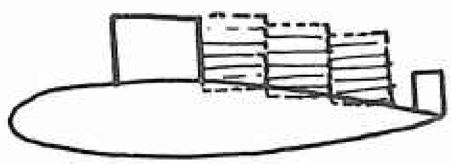


Figure (III.4)

Applying the law of Biot-Savart to a segment of the trailing vortex sheet one obtains for the side wash velocity at the hull centerline:

$$\text{(III.2)} \quad V_{a\,\text{trailers}} = \frac{1}{4\pi}\int_{YB_N}^{YT_N} \frac{(y-y_0)}{(y-y_0)^2+z_0^2}\,\frac{d\Gamma(y)}{dy}\left\{\frac{x_n-x_0}{\left[(x_n-x_0)^2+(y-y_0)^2+z_0^2\right]^{1/2}}+\frac{x_{n-1}-x_0}{\left[(x_{n-1}-x_0)^2+(y-y_0)^2+z_0^2\right]^{1/2}}\right\}dy$$

Where z_0 is the normal distance from the trailing vortex sheet (which may lie in a plane other than $z = 0$) to the point x_0, y_0.

Applying Biot-Savart to the bound element:

$$V_{z\,bound} = -\frac{1}{4\pi}\int_{YH_n}^{YT_n} \Gamma(y)\frac{(x_n - x_0)\quad dy}{\{(x_n - x_0)^2 + (y-y_0)^2 + z_0^2\}^{3/2}} \qquad \text{(III.3)}$$

The total z component of the velocity due to the wake at the instant that the bound element has travelled to $x = x_n$ is:

$$V_{z\,total} = -\frac{1}{4\pi}\int_{YH_n}^{YT_n} \Gamma(y)\frac{(x_n - x_0)\quad dy}{\{(x_n - x_0)^2 + (y-y_0)^2 + z_0^2\}^{3/2}}$$

$$+\frac{1}{4\pi}\sum_{J=1}^{n}\int_{YH_J}^{YT_J}\frac{(y-y_0)}{(y-y_0)^2 + z_0^2}\,\frac{d\Gamma(y)}{dy}\left\{\frac{x_J - x_0}{\{(x_J - x_0)^2 + (y-y_0)^2 + z_0^2\}^{1/2}}\right.$$

$$\left.+\frac{x_{J-1} - x_0}{\{(x_{J-1} - x_0)^2 + (y-y_0)^2 + z_0^2\}^{1/2}}\right\}$$

The other two components of velocity due to the wake, $V_{x\,wake}$, $V_{y\,wake}$ were considered to be negligably small compared to the $V_{z\,total\,wake}$ component.

C. Force and Moment Calculations

The hull of a yawed submarine is typically modeled as a distribution of sources and sinks along the axis of the body to produce the body's axial shape and a distribution of dipoles having their axes pointed in a direction opposing the crossflow to satisfy the crossflow tangency boundary condition. Both of these distributions can experience forces and moments due to the external flow.

Following reference (7), one reaches the conclusion that the source-sink distribution will experience both a force and moment due to the presence of the wake. There will be no force on the dipole distribution since the sail image provides for no influence of the wake velocity field on the local dipole strength required to satisfy the crossflow boundary condition. There is no moment on the dipole distribution due to the wake was a result of the assumption that $V_{y\ wake}$ and $V_{x\ wake}$ are negligably small compared to $V_{z\ wake}$.

The force on the source distribution as a function of distance travelled is calculated from a relationship given by McCreight which is based on slender body theory and Lagally's theorem.

$$Y_{source}^{(1)} = \rho U_\infty \int_{Bow}^{Stern} S'(x) W(x,s)\, dx \qquad \text{(III.5)}$$

Where: W(x, s) is the local sidewash velocity due to the wake and S'(x) is the derivative of the cross sectional area curve for the hull.

Similarly, the moment on the source distribution is given by:

$$N_{sources}(s) = \rho U_\infty \int_{Bow}^{Stern} S'(x) w(x,s)\, x\, dx \qquad (III.6)$$

IV. Rudder-Wake Interaction

A. Following the philosophy described in III. B, the response of the rudder to the velocity field of the sail wake will be obtained by finding the response of the rudder to the wake resulting from a step change in sail circulation, then applying the superposition theorem.

The rudder analysis will be carried out on a pseudo-steady basis; i.e., at each instant of time the rudder an its wake are assumed to be in a steady state condition of vorticity distribution. This is a reasonable assumption. Typically, a submarine rudder has an aspect ratio of roughly the same magnitude as the sail, with a chord measuring about one-third of that of the sail. Hence, the non-dimensional dynamics (transient response of total bound circulation versus number of chord lengths travelled) of the sail and the rudder will be very nearly equal, but the rudder will respond roughly three times as quickly in real time due to its shorter chord length. The pseudo-steady analysis essentially assumes that the rudder responds instantaneously to a change in angle of attack. Since the rudder responds three times as fast as the sail, the pseudo-steady analysis should give a fair representation of the interaction of the sail wake and the rudder.

B. Rudder Response to Step Change in Sail Circulation

The details of all of the steps required to analyze the response of the rudder have been covered in previous sections. The appropriate integral equation is given by II. 10. The equivalent span is calculated from II. 18. The spanwise and chordwise loading modes are those used for the sail (the special chordwise mode is not used). Equation II. 10 is solved by choosing sixteen control points, integrating the loading modes numerically, calculating the left hand side (local angle of attack at control point) from equation III. 4, and solving the resulting system of equations by least-squares. The forces and moments on the rudder are calculated in the same manner as those of the sail, with the major exception that only the steady contributions of each mode are considered.

V. Superposition Theorem

Consider a two-dimensional section of the sail-wake vorticity distribution as shown in Figure V. 1.

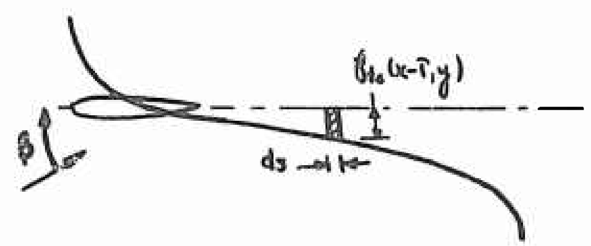


Figure V. 1

The wake can be considered to be constructed from an infinity of wakes from step changes in sail circulation of strength $\gamma_{te}(x-\tau,y)$.

From relationship II. 4:

$$\gamma(x,y,s) = \gamma_{te}(x-\tau,y) = -\frac{\partial}{\partial s}\left(\Gamma_{2D}(x-s,y)\right) \quad (V.1)$$

If, for example, the velocity field, and, hence, the force on the hull due to the wake of a step change in circulation is known, the force on the hull due to the wake of the indicial response of the sail can be calculated from a convolution integral:

$$Y_{HULL}(s) = -\int_0^s Y^*_{HULL}(\tau)\frac{\partial}{\partial\tau}\left\{\Gamma_{2D}(x-\tau,0)\right\}d\tau$$

Where: $Y_{hull}(s)$ = The side force response of the hull to the wake produced by the indicial response of the sail

$Y^*_{hull}(s)$ = The side force response of the hull to the wake produced by a step change in sail circulation

$\Gamma_{2d}(x-\tau, 0)$ = The instantaneous mode strength of the vorticity being shed into the wake

Γ_{2d} typically has very large derivatives at s = 0, so that from a numerical standpoint it is advantageous to integrate V. 2 by parts

$$Y_{H}(s) = -Y^*_{HULL}(\tau)\Gamma_{2D}(s-\tau)\Big|_0^s - \int_0^s -\frac{\partial Y^*_{HULL}}{\partial\tau}(\tau)\Gamma_{2D}(s-\tau)\,d\tau$$

$$= Y^*_{HULL}(0)\,\Gamma_{2D}(s) + \int_0^s \frac{\partial Y^*_{HULL}(\tau)}{\partial\tau}\Gamma_{2D}(s-\tau)\,d\tau \qquad (V.3)$$

Similarly, for the remaining forces and moments on the hull and sail:

$$N_{HULL}(s) = N^{*}_{HULL}(0)\,\Gamma_{20}(s) + \int_0^s \frac{\partial N^{*}_{HULL}(\tau)}{\partial s}\,\Gamma_{20}(s-\tau)\,d\tau \qquad (V.4)$$

$$Y_{RUDDER}(s) = Y^{*}_{RUDDER}(0)\,\Gamma_{20}(s) + \int_0^s \frac{\partial Y^{*}_{RUDDER}(\tau)}{\partial s}\,\Gamma_{20}(s-\tau)\,d\tau \qquad (V.5)$$

$$N_{RUDDER}(s) = N^{*}_{RUDDER}(0)\,\Gamma_{20}(s) + \int_0^s \frac{\partial N^{*}_{RUDDER}(\tau)}{\partial s}\,\Gamma_{20}(s-\tau)\,d\tau \qquad (V.6)$$

$$L'_{RUDDER}(s) = L'^{*}_{RUDDER}(0)\,\Gamma_{20}(s) + \int_0^s \frac{\partial L'^{*}_{RUDDER}(\tau)}{\partial s}\,\Gamma_{20}(s-\tau)\,d\tau \qquad (V.7)$$

VI. Unsteady Response Results for Example Vehicle

A. Vehicle Description

The analyses described in the previous sections were applied to an example hull-sail-rudder combination, the dimensions of which are shown in Figure VI. 1. For simplicity of calculation, the sail and rudder planforms were both chosen to be rectangular.

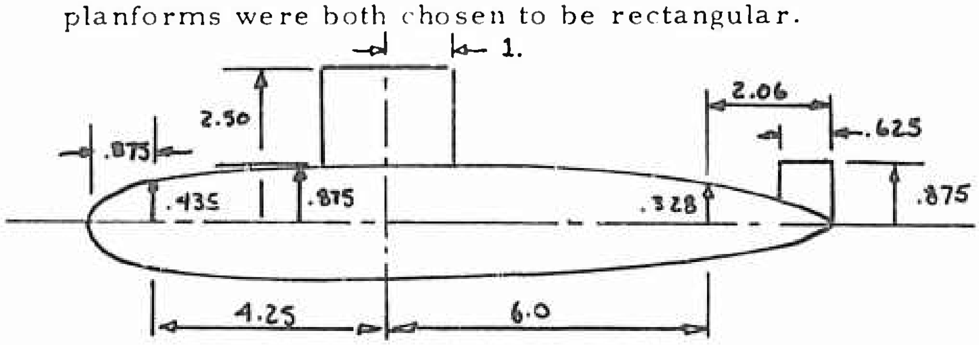


Figure (VI. 1)

The bow and stem portions of the hull were assumed to be parabolic ($r \approx K\sqrt{x-c}$) while the remainder of the hull was approximated by a second order interpolating polynomial ($r \approx a + bx + cx^2$).

B. Indicial Response of the Sail

The strengths of the vorticity cross modes used in the solution of the equation (II. 6) versus the number of chord lengths travelled after the step change in sideslip angle are plotted in Figure VI. 2. For this aspect ratio, the circulation buildup is very rapid; after the sail has travelled only one chord length, the lifting cross mode C_{11} has risen to roughly 85% of its steady state value. A rectangular wing of aspect ratio 6 would have to travel roughly 4 half chords to attain a similar percentage of its steady state circulation.

The forces and moments corresponding to the circulation responses of Figure VI. 2 are given in Figure VI. 3. a, b and c. The force and moment calculations indicate that at the starting instant the sectional forces on the sail are increasing, producing the initial "humps" in the responses. The presence of the "humps" is a significant result since, for example, the sail will experience a maximum roll moment overshoot of 28% during the transient response of the sail.

C. Response of the Hull to a Step Change in Sideslip Angle (Sail-Hull Interaction Only)

The side force and yaw moment on the hull due to a step change in sideslip angle are plotted in Figures VI. 4. a and b. Near the start of the response, both the side force and yaw moment are positive, whereas their corresponding steady state values are negative. This effect is due to extensive changes in the velocity field due to the wake that occur as the wake lengthens. Near the start of the response the wake has a velocity (V_z) distribution along the axis of the hull as shown in Figure VI. 5. a); the steady state distribution of V_z is shown in Figure VI. 5. b.

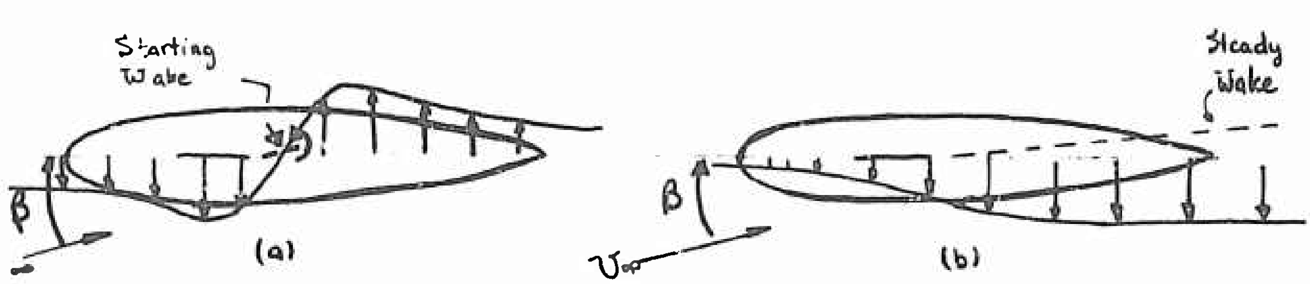


Figure (VI.5)

The initial distribution of wake vorticity produces a negative V_z on the forward part of the hull (source distribution) and a positive V_z on the after part of the hull (sink distribution) and hence, by Lagally's theorem, produces a positive side force. The steady wake's predominant effect is to produce a negative velocity on the after part of the hull and, by Lagally's theorem, a negative side force.

D. Response of Rudder to Step Change in Sideslip Angle

The side force, yaw moment, and roll moment on the rudder due to the sail wake are shown in Figures VII. 6. a, b and c). The wake has a comparatively small influence on the rudder until the trailing edge of the wake passes by the rudder. As the trailing edge of the wake passes the rudder the forces and moments change sign (indicating a change in sign of the sidewash produced by the wake at the rudder) and rise rapidly toward their final values.

E. Total Configuration Response

In order to compare the relative magnitudes of the forces developed on the sail, hull, and rudder and the relative time scales involved in their responses Figures VI. 7. a, b and c were constructed. These plots represent the sums of the total transient forces and moments on the sail but only the interaction forces and moments due to the sail wake on the hull and rudder. Since the sail has the most dominant contribution to the force and moment on the total configuration, reference lines for the steady state forces and moments on the sail were added to Figures VI. 7. a, b and c

to facilitate a quantitative comparison of the importance of each interaction. The yaw moment was referred to the mid-chord of the sail; the roll moment was taken about the axis of symmetry of the hull.

In all cases (side force, yaw moment, roll moment) the initial portion of the response is dominated by the sail response. The "hump" that was mentioned in Section VI. B indicates that initially the submarine acts as a lightly damped force and moment generator; i.e., a step input of sideslip angle causes a rapid rise- high overshoot response of force and moment.

The portion of the response between one and five chord lengths travelled is characterized by the interaction response of the hull. The side force response amounts to onlv a small percentage of the steady force on the sail. The interaction yaw moment of the hull, however, causes an ultimate moment variation of 20% of the steady moment of the sail.

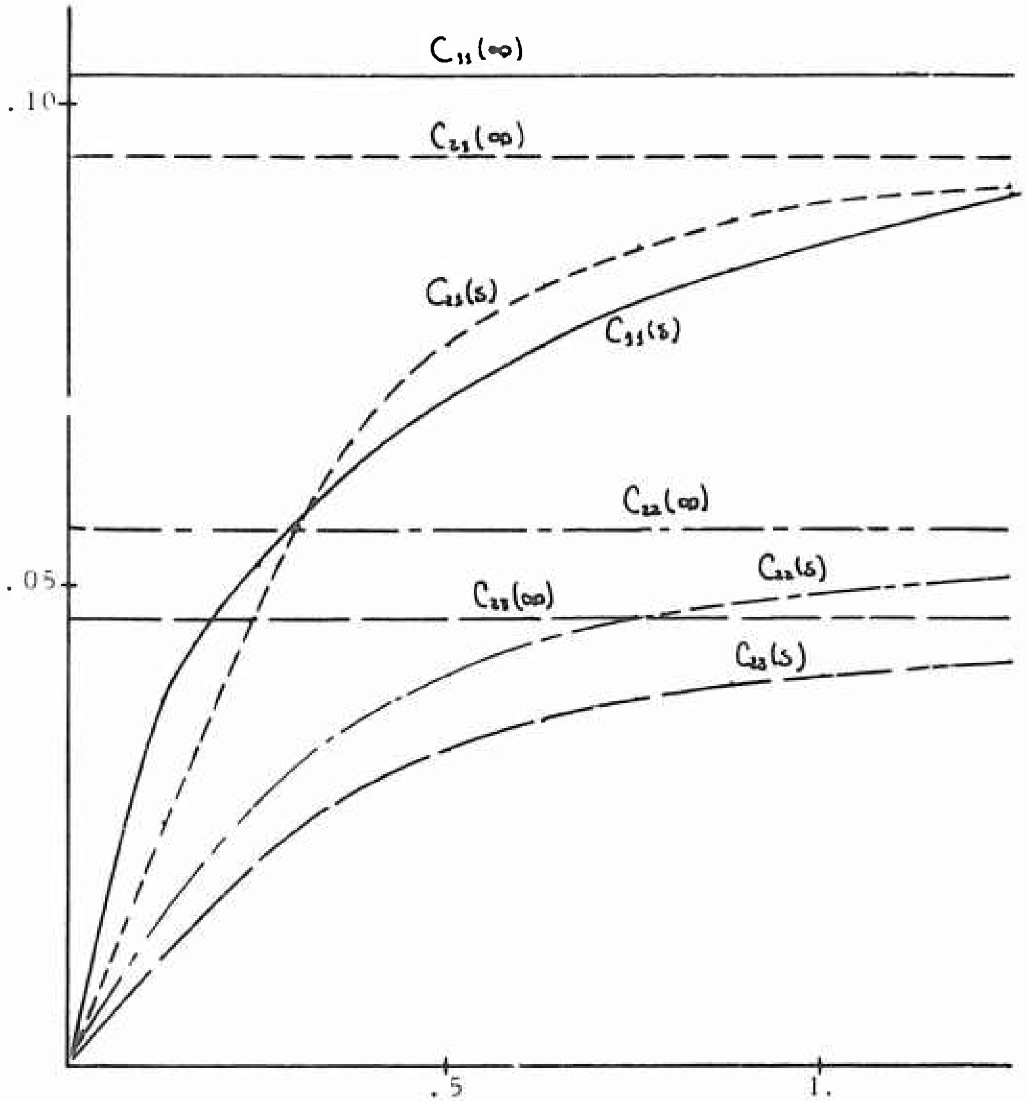
The portion of the total configuration response past five chord lengths is characterized by an abrupt change in force and moment due to the rudder-wake interaction. The ultimate change in side force caused by the rudder-wake interaction is roughly 20% of the steady force on the sail. The yaw moment response is the most dramatic, the change caused by the rudder being roughly 120% of the steady moment on the sail. This is a result, as one might expect, of the large moment arm between the mid-chord of the sail and the center-of-pressure of the rudder. The roll

moment interaction of rudder amounts to a very small percentage of the steady roll moment on the sail since the rudder's center-of-pressure is comparatively close to the axis of the hull.

Figure VI.2

Circulation Mode Responses of Sail to .1 Radian Step Change in Sideslip Angle

$A\!\!\!R_{eq} = 1.097 \quad \Theta = .05$



Distance Traveled-Halfchords of Sail

Figure VI. 3a

Side Force Response of Sail to .1rad Step Change in Sideslip Angle

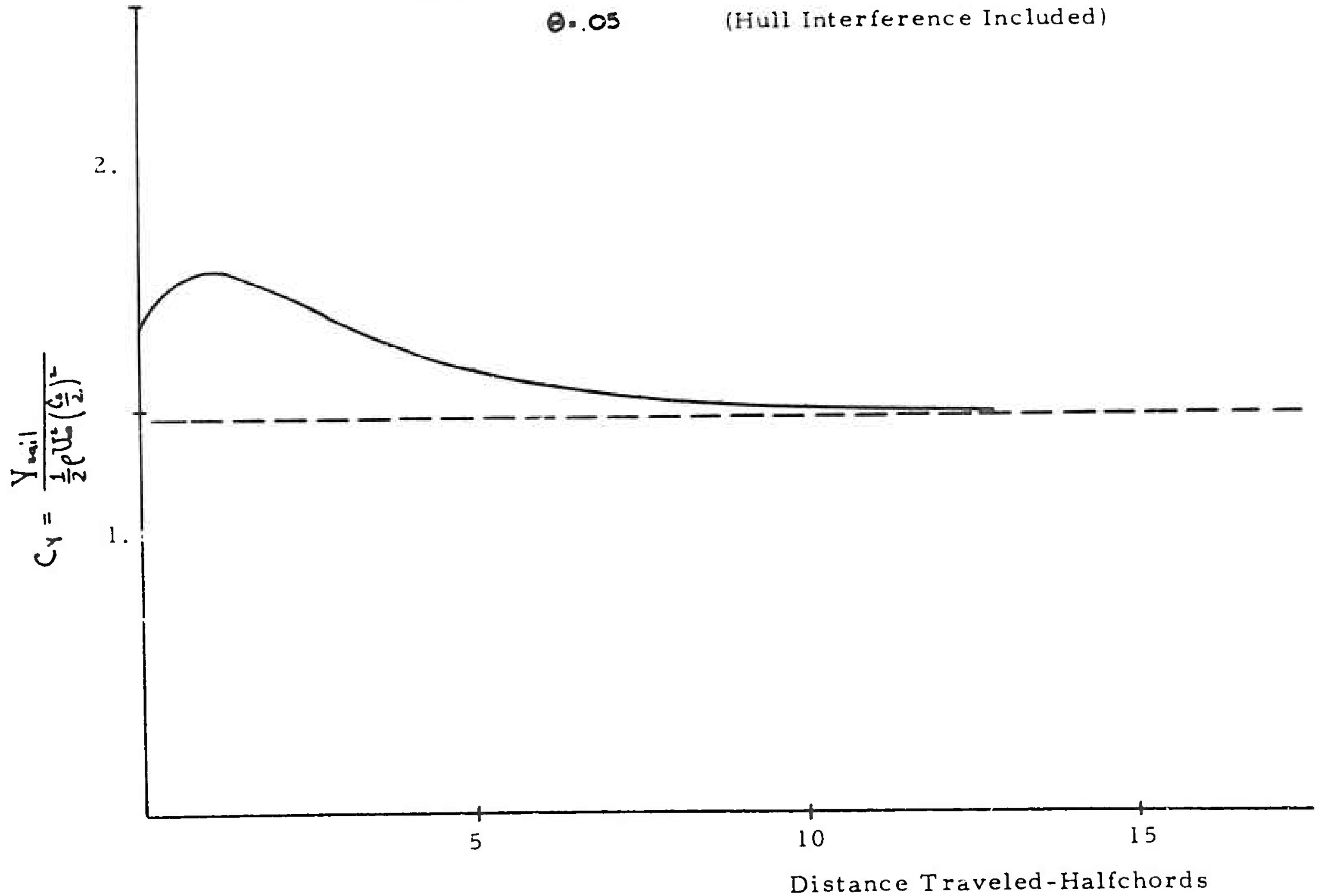


Figure VI. 3b

Yaw Moment Response of Sail to . 1rad Step Change in Sideslip Angle

$\Theta = .05$ (Hull Interference Included)

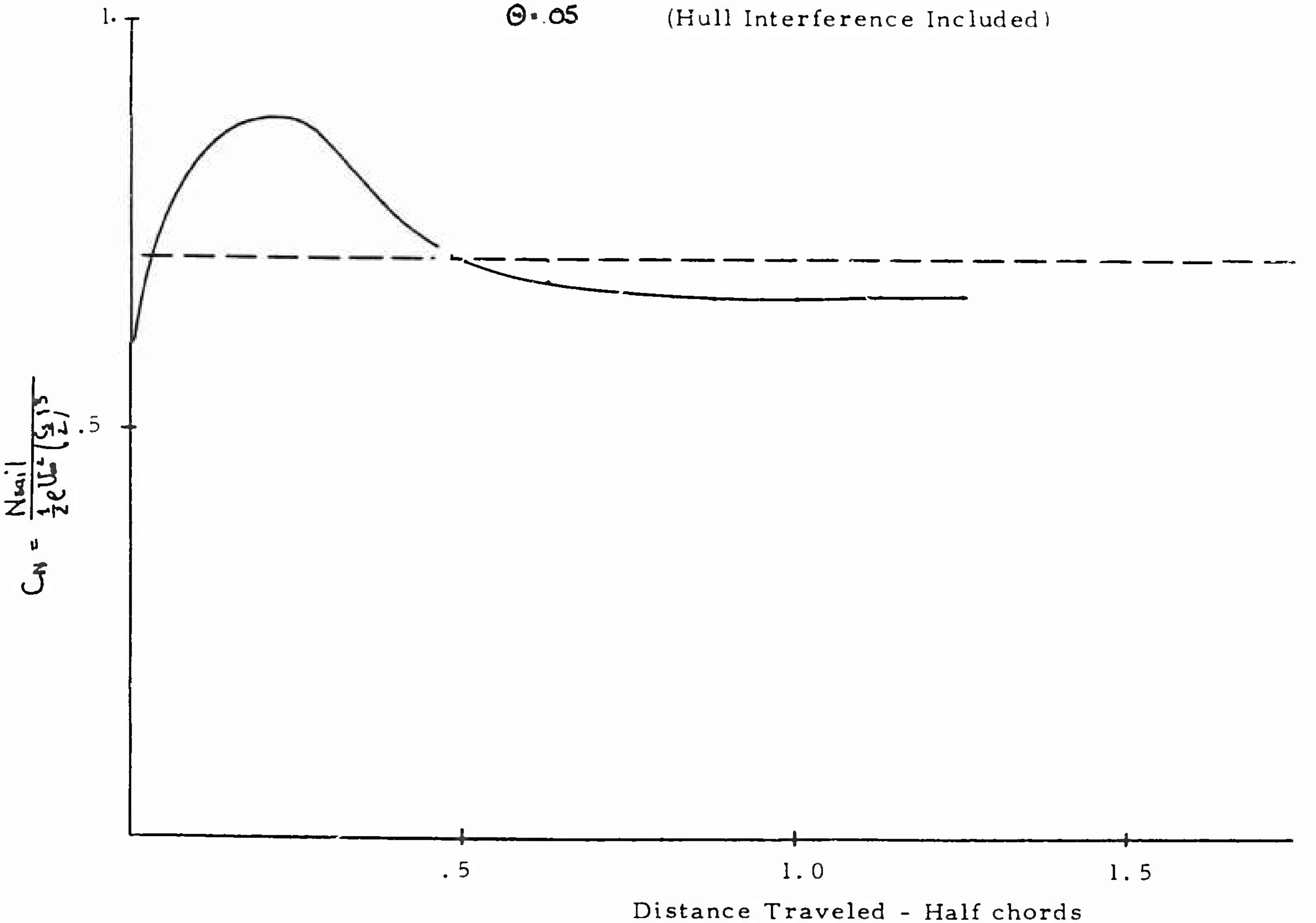


Figure VI. 3c

Roll Moment Response of Sail to .1rad Step Change in Sideslip Angle

$\Theta = .05$ (Hull Interaction Included)

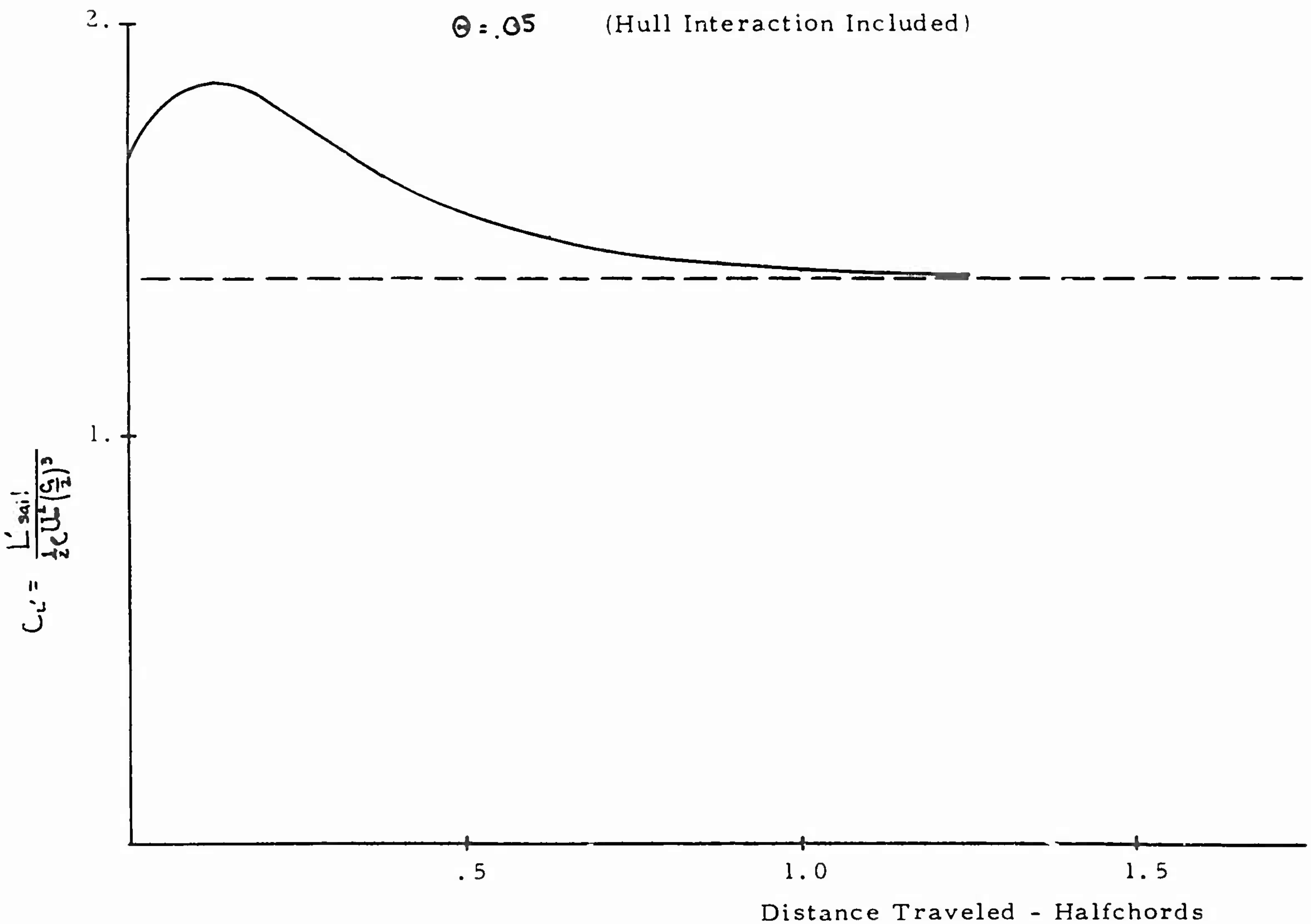


Figure VI.4a

Side Force Response of Hull to .1radian Step Change in Sideslip Angle

(Hull-Sail Interaction Only)

$\beta = .1$, $\Theta = .05$

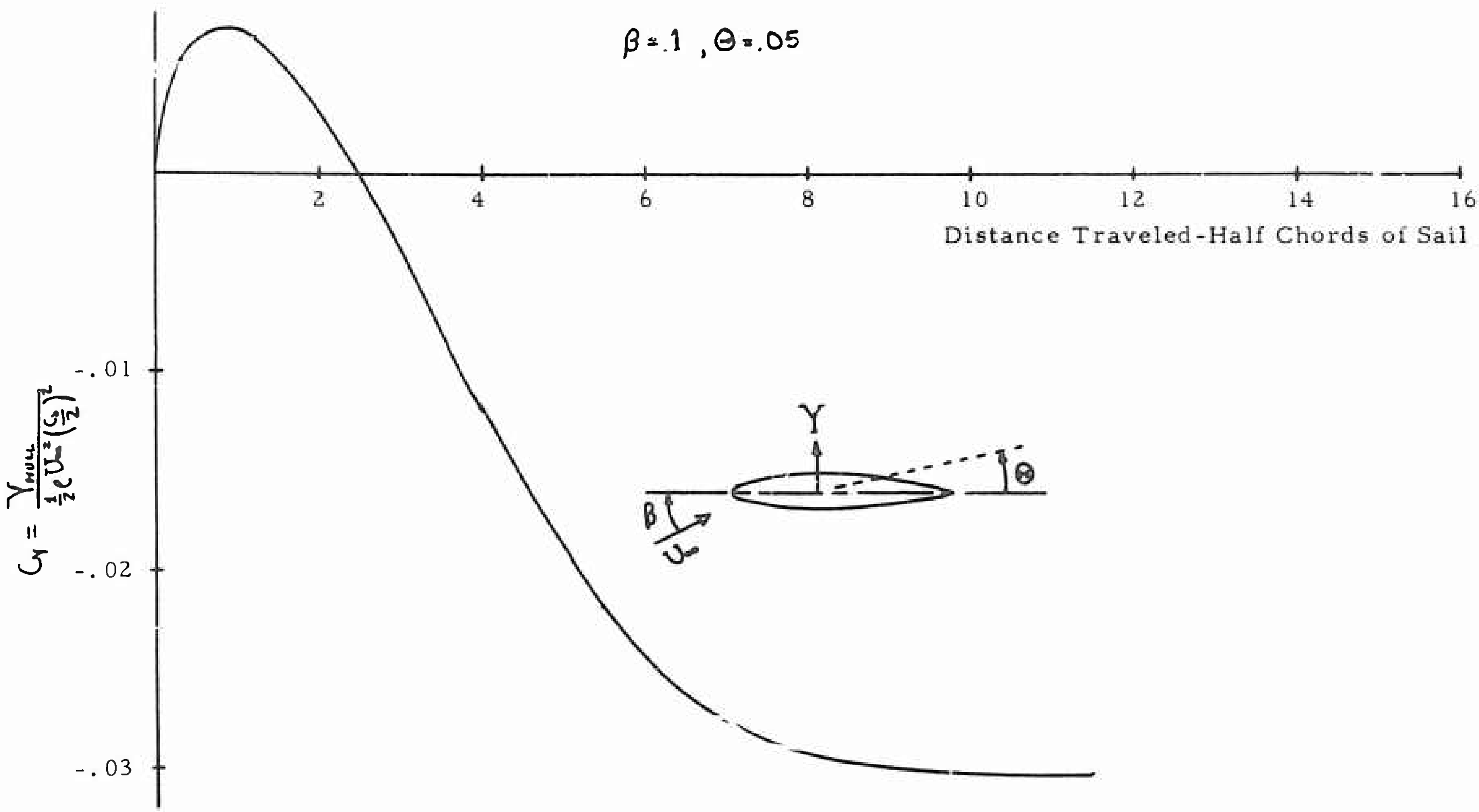


Figure IV.4b

Yaw Moment Response of Hull to .1 radian Change in Sideslip Angle

(Hull-Sail Interaction Only)

$\beta = .1 \quad , \Theta = .05$

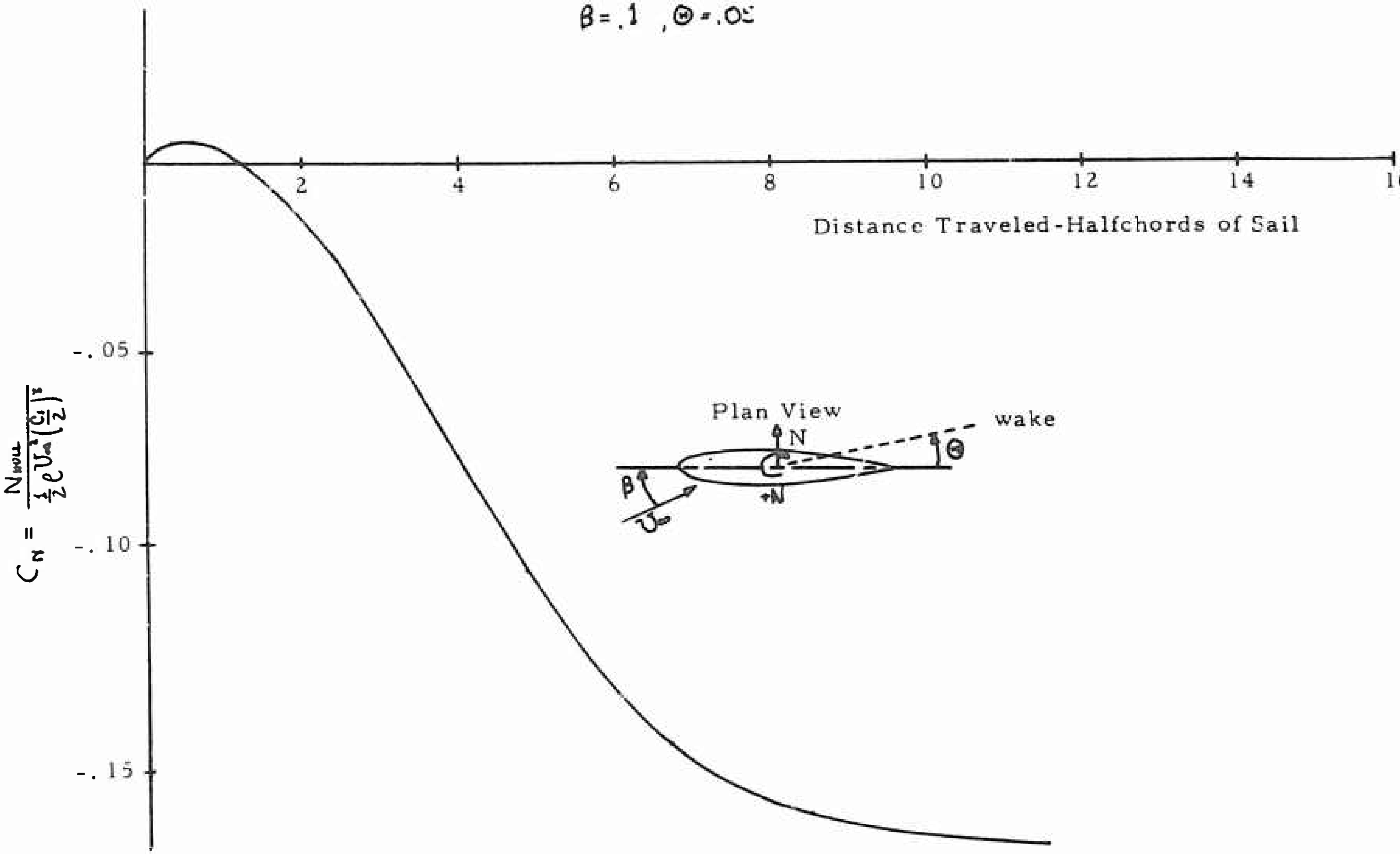


Figure VI. 6a

Side Force Response of Rudder to . l radian Step Change in Sideslip Angle

(Sail-Rudder Interaction Only)

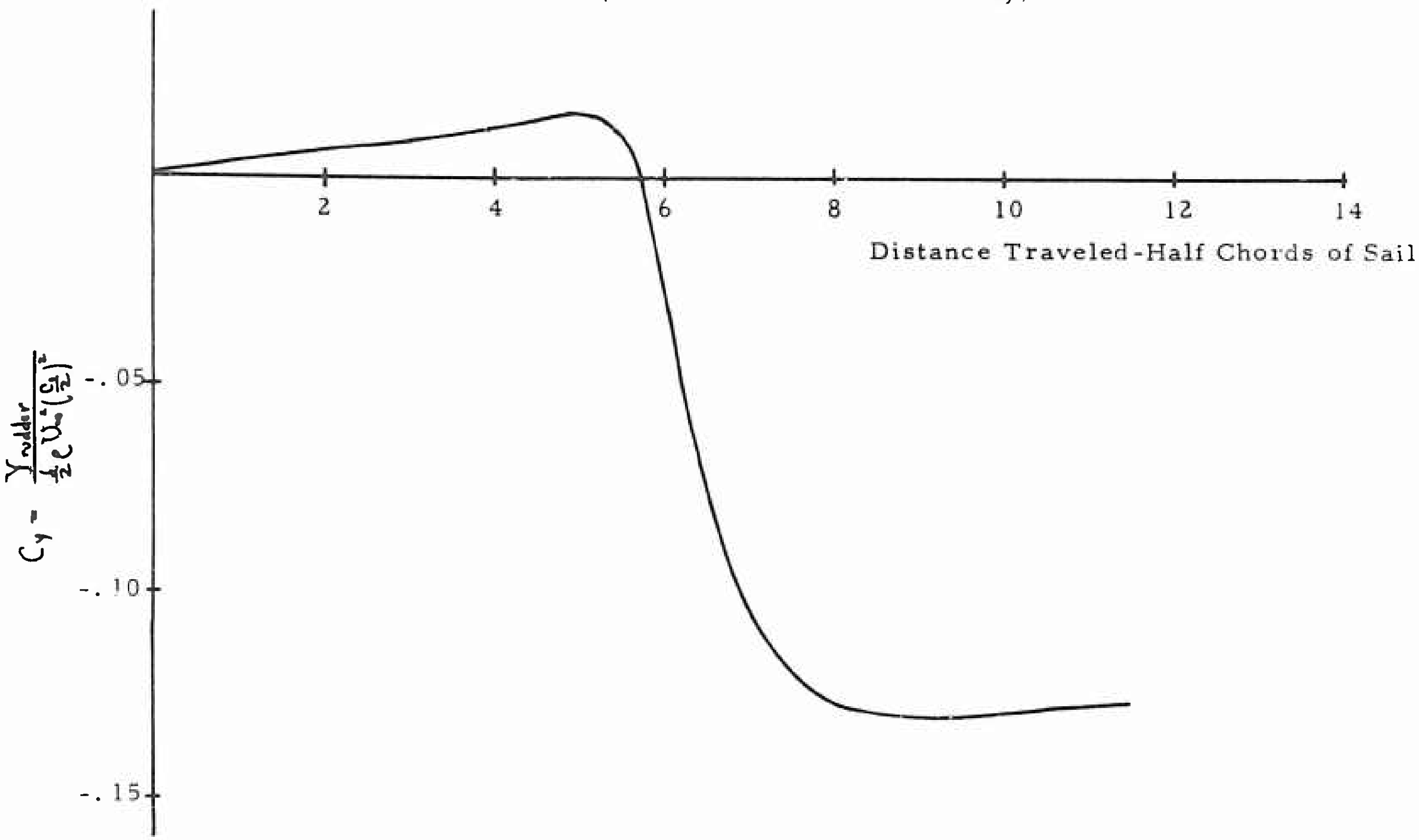


Figure VI.6b

Yaw Moment Response of Rudder to Step Change in Sideslip Angle

(Sail-Rudder Interaction Only)

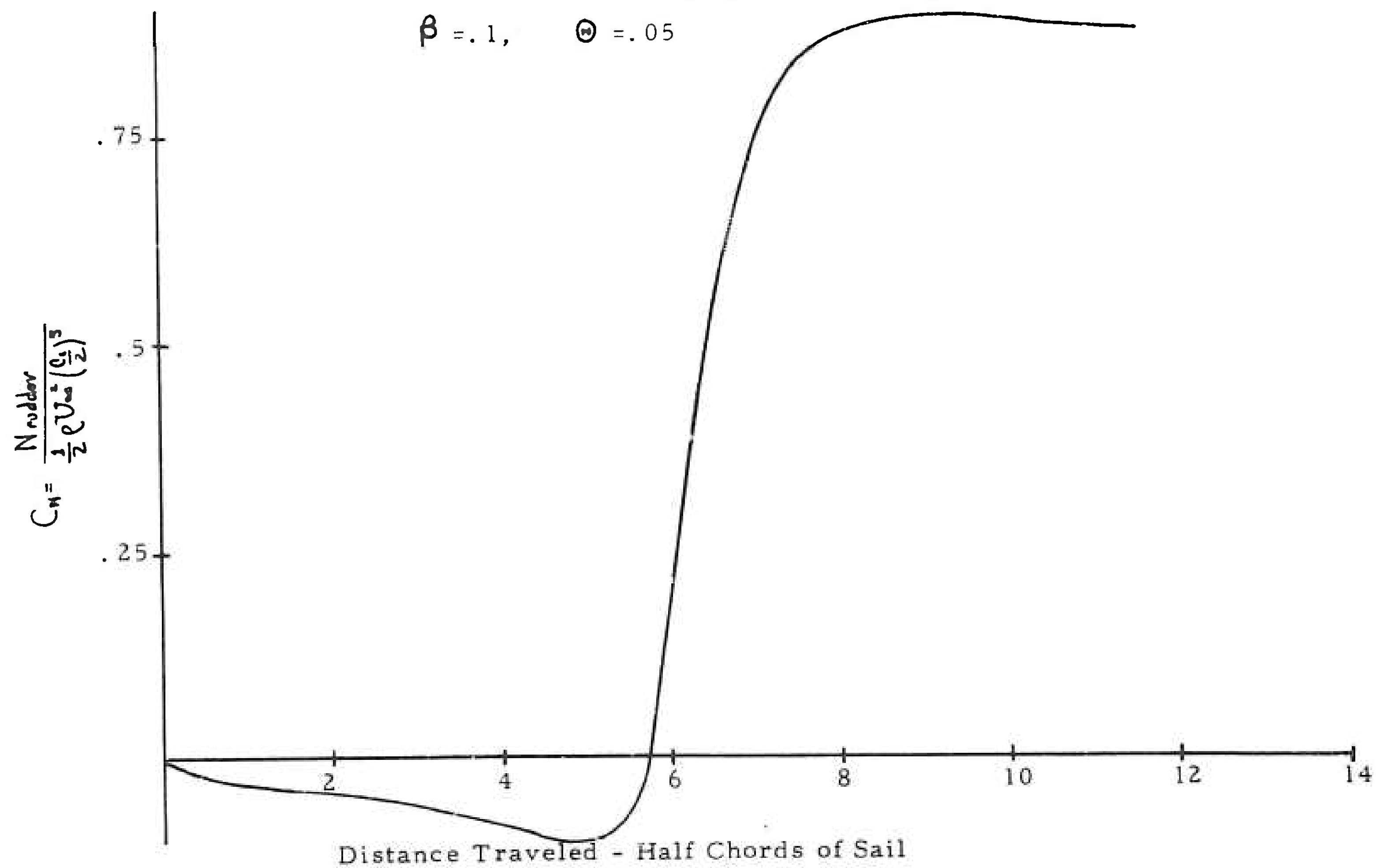


Figure VII.6c

Roll Moment Response of Rudder to .1 radiam Step Change in Sideslip Angle

$\beta = .1$, $\Theta = .05$

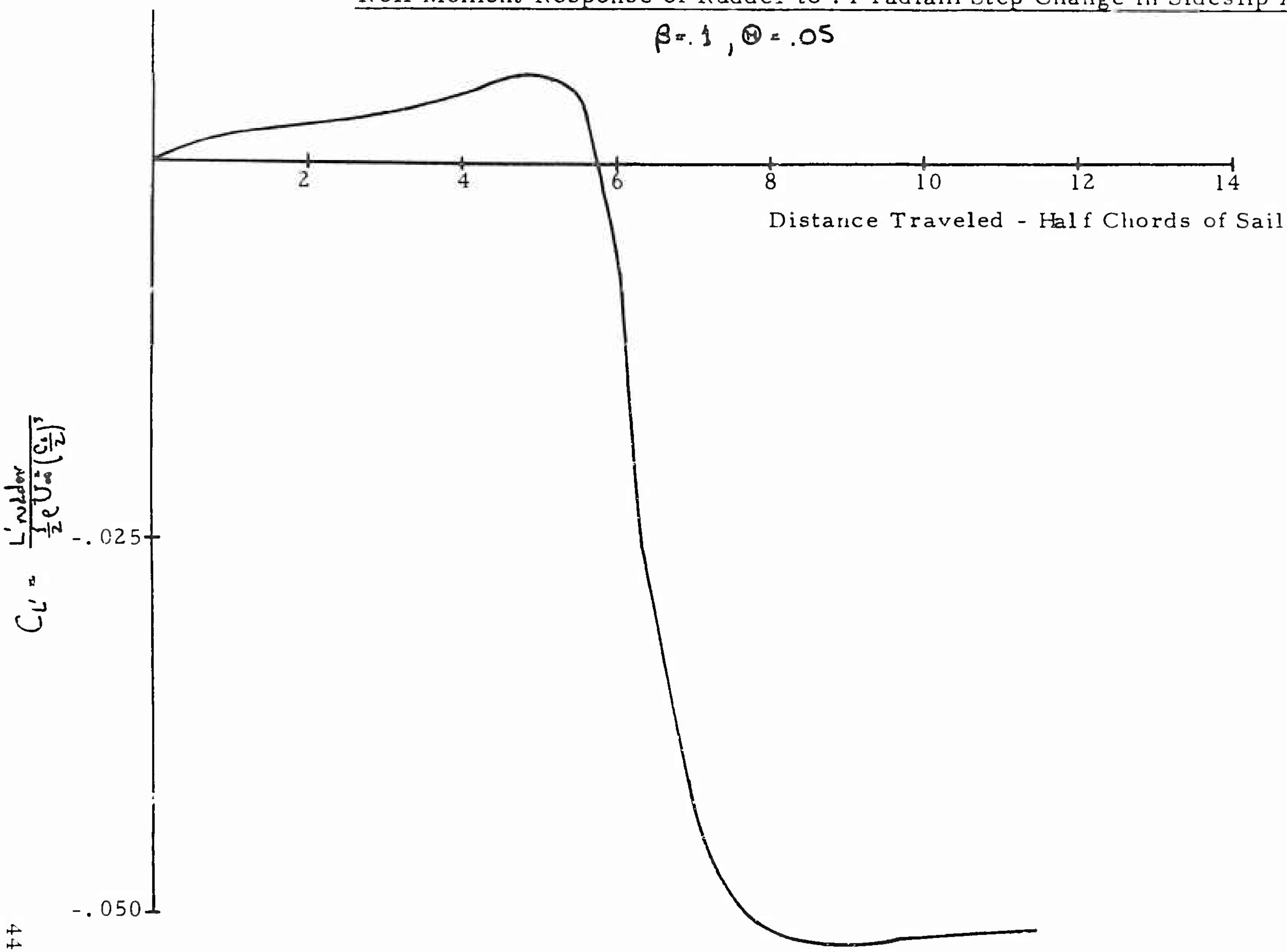


Figure VI. 7a

Side Force Response of Submarine Hull-Sail-Rudder Combination to . 1rad Step Change in Sideslip Angle

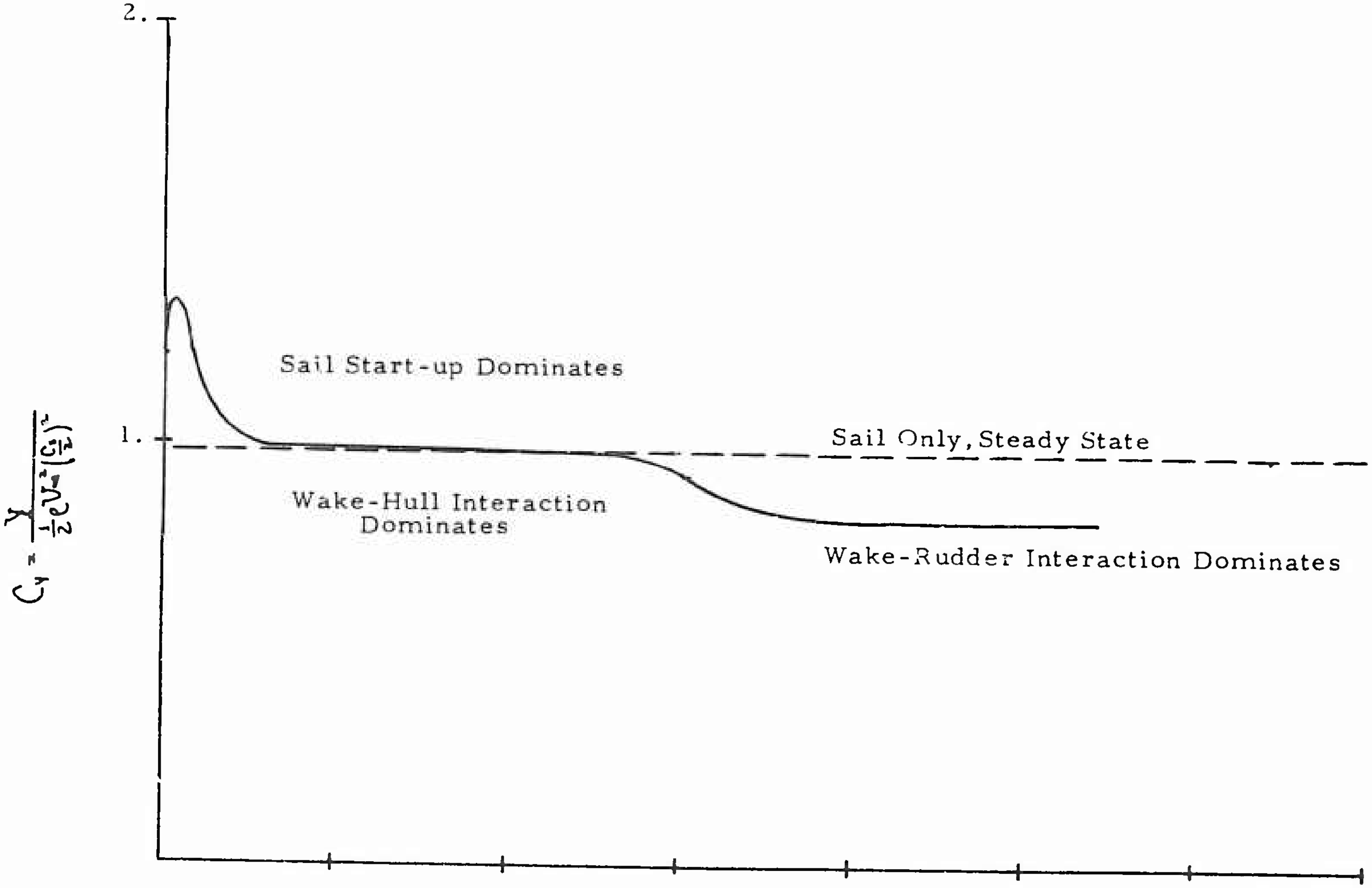


Figure VI.7b

Yaw Moment Response of Hull-Rudder Combination to Step Change in Sideslip Angle

$\beta = .1$, $\Theta = .05$

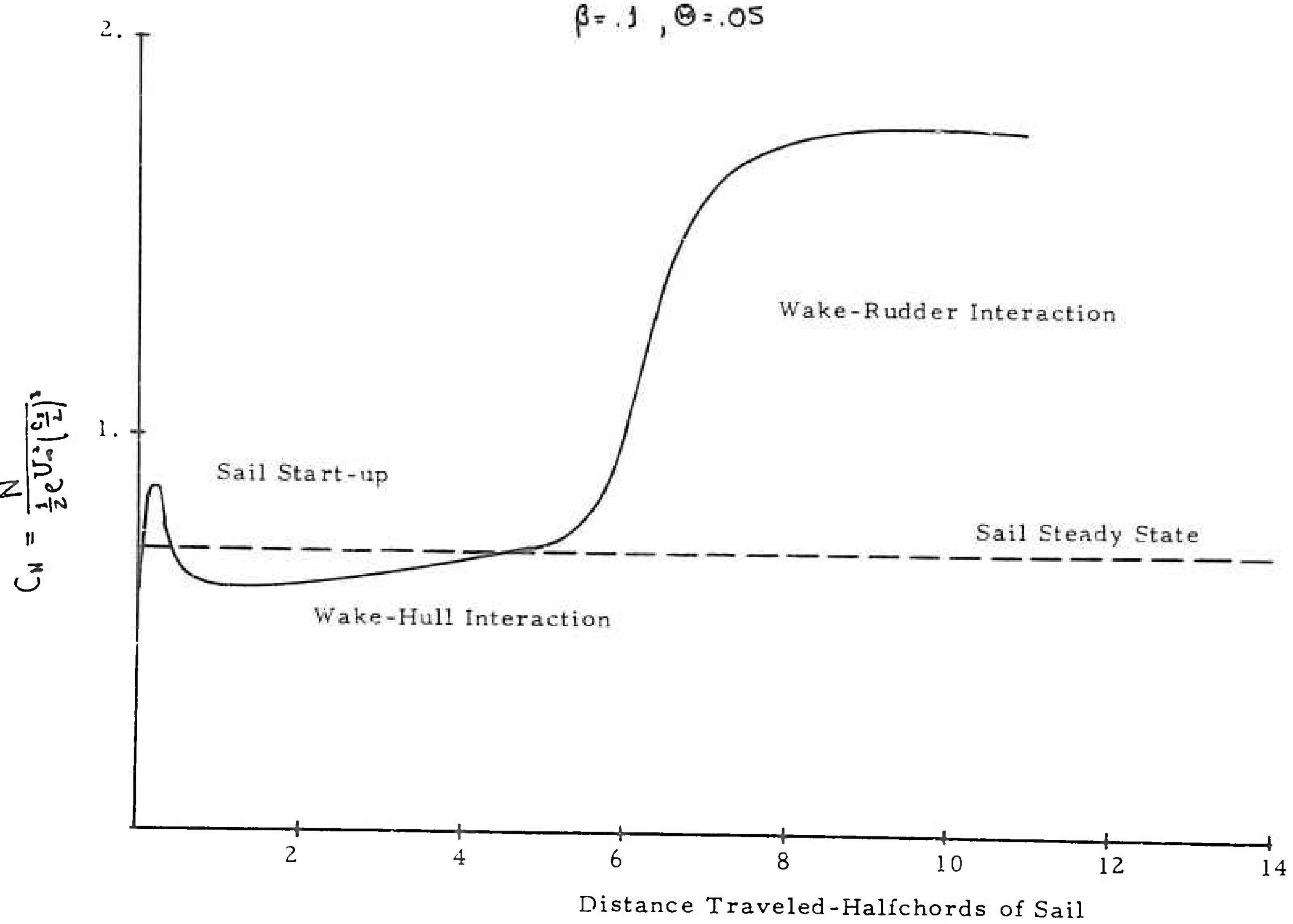
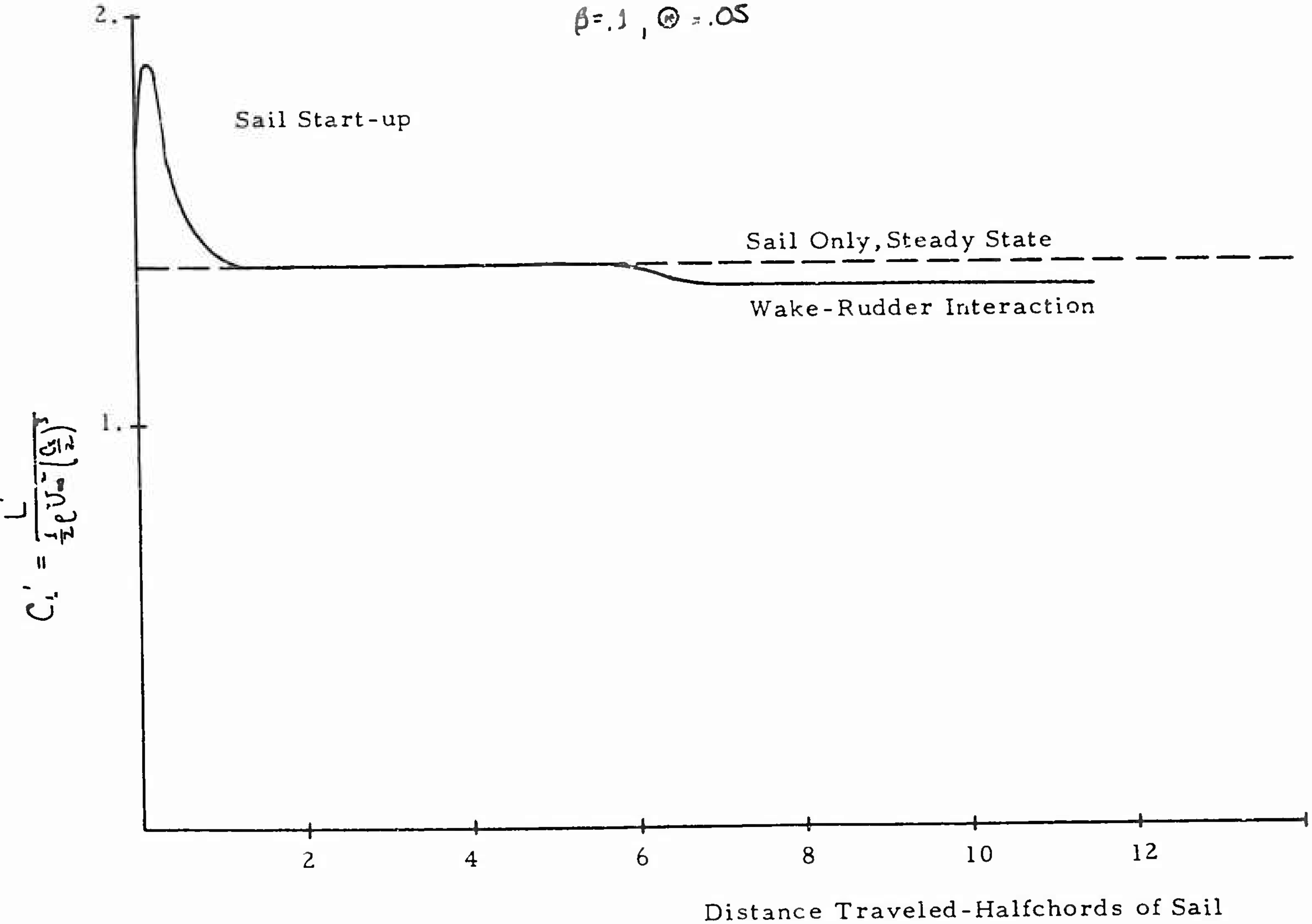


Figure VI. 7c

Roll Moment Response of Submarine Hull-Sail-Rudder Combination to .1 rad Step Change in Sideslip Angle (Moment taken about hull axis of symmetry)

$\beta = .1$, $\Theta = .05$



VII. Conclusions

1) The transient response of the sail to a step change in sideslip angle shows a large initial overshoot of sideforce, yaw moment, and roll moment with respect to the steady state values.

2) The interaction response of the hull to the developing sail wake shows the development of a modest sideforce and a more sizeable stabilizing yaw moment.

3) The interaction response of the rudder to the developing sail wake is characterized by a sudden reversal of rudder side force as the trailing edge of the sail wake passes by the rudder. This force reversal yeilds a small change in roll moment and a very large change in yaw moment.

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APPENDIX A

DERIVATION OF EQUATION II. 1

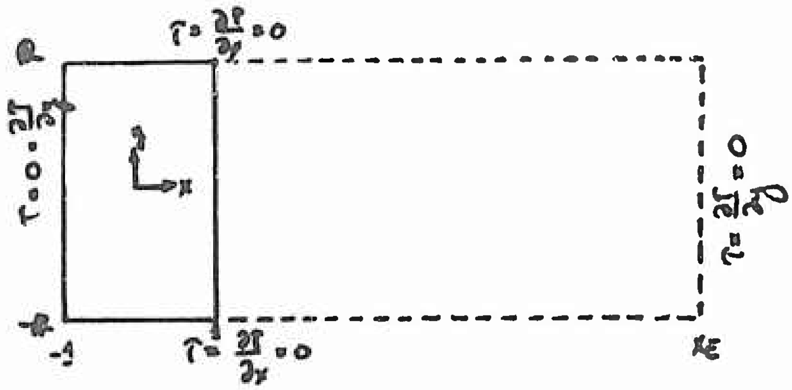
For a distribution of dipoles:

$$w(x_0, y_0, z_0) = \frac{-1}{4\pi} \frac{\partial^2}{\partial z_0^2} \iint\limits_{\substack{\text{Sail}\\+\\\text{Wake}}} \frac{\tau(x,y)\,dx\,dy}{\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}}$$

Since this flow satisfies Laplace's equation, the integral can be written:

$$w(x_0, y_0, z_0) = \frac{1}{4\pi} \iint\limits_{\substack{\text{Sail}\\+\\\text{Wake}}} \tau(x,y)\left\{\frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right)\right\} dx\,dy$$

Considering a rectangular wing, the boundary conditions on τ are given below:



Integrating the first term by parts in the x direction:

$$\underset{\substack{\text{Sail}\\ +\\ \text{Wake}}}{\oiint} \Gamma \frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) dx\,dy = \int_{-R}^{R} dy \int_{-1}^{x_E} \Gamma \frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) dx$$

$$= \int_{-R}^{R} dy \left[\left\{ \Gamma \frac{\partial}{\partial x}\left(\frac{1}{r}\right) \right\}_{-1}^{x_E} - \int_{-1}^{x_E} \frac{\partial \Gamma}{\partial x} \frac{\partial}{\partial x}\left(\frac{1}{r}\right) dx \right]$$

$$= -\int_{-R}^{R} dy \int_{-1}^{x_E} \frac{\partial \Gamma}{\partial x} \frac{\partial}{\partial x}\left(\frac{1}{r}\right) dx$$

Integrating by parts in the y direction:

$$-\int_{-R}^{R} dy \int_{-1}^{x_E} \frac{\partial \Gamma}{\partial x} \frac{\partial}{\partial x}\left(\frac{1}{r}\right) dx = -\int_{-1}^{x_E} dx \left\{ -\left[\frac{\partial \Gamma}{\partial x} \frac{(x-x_0)(y-y_0)}{\{(x-x_0)^2 + z_0^2\} r} \right]_{-R}^{R} + \int_{-R}^{R} \frac{\partial^2 \Gamma}{\partial x \partial y} \frac{(x-x_0)(y-y_0)}{\{(x-x_0)^2 + z_0^2\} r} dy \right\}$$

$$= -\int_{-1}^{x_E} dx \int_{-R}^{R} \frac{\partial^2 \Gamma}{\partial x \partial y} \frac{(x-x_0)(y-y_0)}{\{(x-x_0)^2 + z_0^2\} r} dy$$

Hence:

$$\frac{1}{4\pi} \underset{\substack{\text{Sail}\\ +\\ \text{Wake}}}{\oiint} \Gamma(x,y) \frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) dx\,dy = -\frac{1}{4\pi} \int_{-R}^{R} dy \int_{-1}^{x_E} \frac{\partial^2 \Gamma}{\partial x \partial y} \frac{(x-x_0)(y-y_0)}{\{(x-x_0)^2 + z_0^2\} r} dx$$

A similar integration by parts procedure yields:

$$\frac{1}{4\pi}\oiint_{\substack{\text{Sail}\\+\\\text{Wake}}} \gamma(x,y)\frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right)dx\,dy = \frac{-1}{4\pi}\int_{-R}^{R}dy\int_{-1}^{x_E}\frac{\partial^2\gamma}{\partial x\,\partial y}\frac{(x-x_0)(y-y_0)}{[(y-y_0)^2+z_0^2]\,r}\,dx$$

Hence:

$$w(x_0,y_0,z_0) = \frac{1}{4\pi}\oiint_{\substack{\text{Sail}\\+\\\text{Wake}}} \gamma(x,y)\left\{\frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right)\right\}dx\,dy$$

$$= \frac{-1}{4\pi}\int_{-R}^{R}dy\int_{-1}^{x_E}\frac{\partial^2\gamma}{\partial x\,\partial y}\left\{\frac{1}{(x-x_0)^2+z_0^2}+\frac{1}{(y-y_0)^2+z_0^2}\right\}\cdot\frac{(x-x_0)(y-y_0)}{\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}}\,dx$$

APPENDIX B. COMPUTER PROGRAM LISTINGS

SAIL STEP RESPONSE PROGRAM

INPUT: 1) Vehicle Geometry

2) Side slip Angle

3) Trailing Vortex Sheet Angle

OUTPUT: 1) Step by Step Circulation on Sail

2) Step by Step Forces and Moments on Sail

3) Exponential Approximations to Circulation Response of Sail

```
      DIMENSION CP(20,2),DWS(20,13),DWS1(20,13),B(20),B1(12),CUU(5,40)
      DIMENSION CWJ(3,40),WGF(3,20,40),IL(20),WWS(20,3),CC(12),TAU(12,2)
      DIMENSION A(12)
      DIMENSION USF(12,20),USM(12,20),USR(12,20),TSF(20),TYM(20),TRM(20)
      DIMENSION ST(20,13),SF(5),RM(5)
      READ100,NMC,NMS,NCPC,NCPS,NSP,H,AL,TH,RHS,RSL
      NCP=NCPC*NCPS
      NSPS=NSP
  100 FORMAT(5I5,5F5.3)
      READ200,(CP(I,1),I=1,NCP)
  200 FORMAT(16F5.3)
      NMDS=NMS*NMC
C        CALCULATE EQUIVALENT SPAN & JUNCTION COORDINATE
      AR=.5*(RSL-RHS)*(RSL+RHS)/RSL
      Y1=AR-RSL+RHS
      YH=.6366197*ARSIN(Y1/AR)
C        ESTABLISH SPANWISE CONTROL POINTS
      DO 20 I=1,NCPC
   20 CP(1+(I-1)*NCPS,2)=Y1+.1*AR
      DO 21 I=1,NCPC
      DO 21 J=2,NCPS
   21 CP(J+(I-1)*NCPS,2)=(2*J-3)*.5*AR/(NCPS-1)
C        USE ZEROETH MODE TO SOLVE BC ON FIRST STEP
      H1=.5*H
      H2=1.+H1
      H3=1.-H1
      AR1=AR/H2
      DO 1 I=1,NCP
      X0=(CP(I,1)-H1)/H2
      Y0=CP(I,2)/H2
      DWS1(I,NMDS+1)=AL*(1.+(RHS/(RSL-AR+CP(I,2)))**2)
      DO 1 J=1,NMS
      YH1=YH/H2
      WWS(I,J)=SINT(X0,Y0,J,0,AR1,YH1)/H2
    1 DWS1(I,J)=WWS(I,J)
      DO 15 I=1,NCP
      X0=CP(I,1)
      Y0=CP(I,2)
      DO 15 IJ=2,NMC
      K=(IJ-1)*NMS
      DO 15 J=1,NMS
   15 DWS1(I,K+J)=SINT(X0,Y0,J,IJ,AR,YH)
      N2=NMDS+1
      PRINT50
      DO 30 I=1,NCP
   30 PRINT500,(DWS1(I,J),J=1,N2)
      CALL GLSQ(DWS1,B1,IL,NCP,NMDS,BUG,0.,0.)
C        FIND TRAILING EDGE VALUE OF VORTICITY,ETC.
      DIP=SQRT(1.-(H3/H2)**2)
      VOR=H3/H2/H2/DIP
```

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```
      DO 2 J=1,NMS
      COU(J,1)=-1.*B1(J)*DIP/3.1416
    2 CWJ(J,1)=-1.*B1(J)*VOR
      NO=NMS+1
      DO 17 I=NO,NMCS
   17 COU(I,1)=B1(I)
C        FIND DOWNWASH OF UNIT PSEUDO MODE
      DO 3 I=1,NCP
      XO=CP(I,1)
      YO=CP(I,2)
      DO 3 J=1,NMS
    3 WWS(I,J)=(WWS(I,J)+(DIP/3.1416)*(SINT(XO,YO,J,1,AR,YH)-TECMC(AR,H,
     11,XO,YO,J,TH,YH)))/VOR
      DO 4 I=1,NCP
    4 PRINT500,(WWS(I,J),J=1,NMS)
  500 FORMAT('0',10X,10E12.4)
      DO 5 K=1,NMS
      DO 5 I=1,NCP
      XO=CP(I,1)
      YO=CP(I,2)
      DO 5 J=1,NSD
    5 WGF(K,I,J)=WAKE(H,AR,J,XO,YO,K,TH,YH)
C        FIX MATRIX EQUATION ARRAY FOR SUBSEQUENT VALUES OF 1ST MODE
      DO 6 I=1,NCP
      XO=CP(I,1)
      YO=CP(I,2)
      DO 6 J=1,NMS
    6 DWS(I,J)=SINT(XO,YO,J,1,AR,YH)-WWS(I,J)*6.2832/H
      IF(NMC.EQ.1)GO TO 13
      DO 12 I=1,NCP
      XO=CP(I,1)
      YO=CP(I,2)
      DO 12 J=2,NMC
      N=NMS
      DO 12 K=1,NMS
      L=N*(J-1)+K
   12 DWS(I,L)=SINT(XO,YO,K,J,AR,YH)
   13 CONTINUE
      DO 11 N=2,NSD
      DO 7 I=1,NCP
      B(I)=0.
      XO=CP(I,1)
      YO=CP(I,2)
      DO 7 J=1,NMS
C        FIND NEW DOWNWASH OF TRAILING EDGE MODE
      B(I)=B(I)+COU(J,1)*TECMC(AR,H,N,XO,YO,J,TH,YH)
C       FIND DOWNWASH DUE TO PL MODES IN WAKE
      N1=N-1
      DO 8 IJ=1,N1
    8 B(I)=B(I)+CWJ(J,IJ)*WGF(J,I,N-IJ)
```

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```
    7 B(I)=B(I)-(CWJ(J,N-1)+6.2832*COU(J,N-1)/H)*WWS(I,J)
      DO 9 I=1,NCP
      DWS1(I,NMDS+1)=B(I)+AL*(1.+(RHS/(RSL-AR+CP(I,2))))**2)
      DO 9 J=1,NMDS
    9 DWS1(I,J)=DWS(I,J)
      CALL GLSQ(DWS1,B1,IL,NCP,NMDS BUG,0.,0.)
      PRINT50
      N3=NMDS+2
      DO 31 I=1,NCP
      DWS(I,N3)=DWS(I,1)*B1(1)
      DWS(I,N2)=B(I)+AL*(1.+(RHS/(RSL-AR+CP(I,2))))**2)
      DO 32 J=2,NMDS
   32 DWS(I,N3)=DWS(I,N3)+B1(J)*DWS(I,J)
   31 PRINT500,(DWS(I,J),J=1,N3)
      DO 25 I=1,NMDS
   25 COU(I,N)=B1(I)
      DO 11 J=1,NMS
   11 CWJ(J,N)=(6.2832/H)*(COU(J,N)-COU(J,N-1))-CWJ(J,N-1)
      DO 23 I=1,NCP
      X0=CP(I,1)
      Y0=CP(I,2)
      ST(I,NMS+1)=AL*(1.+(RHS/(RSL-AR+Y0)))**2)
      DO 23 J=1,NMS
   23 ST(I,J)=SINT(X0,Y0,J,0,AR,YH)
      CALL GLSQ(ST,B1,IL,NCP,NMS,BUG,0.,0.)
      CALL SR(AR,YH,NMS,SF,RM)
      PRINT800,AR
  800 FORMAT('1',10X,'STEP RESPONSE OF SUBMARINE SAIL WITH HULL INTERFER
     1ENCE,AR= ',F5.3)
      PRINT400,H
  400 FORMAT('0',10X,'BETA= .1  H=',F6.4,' HALF CHORDS')
      PRINT625
  625 FORMAT('0',10X,'STARTING MODE STRENGTHS')
      II=0
      PRINT600,(II,I,B1(I),I=1,NMS)
      DO 10 N=1,NSD
      PRINT300,N
  300 FORMAT('0',10X,'WAKE LENGTH=',I3,2X,'STEPS')
      DO 10 I=1,NMC
   10 PRINT600,(I,J,COU((I-1)*NMS+J,N),J=1,NMS)
  600 FORMAT(1X,3(2X,'B(',I1,'-',I1,')=',E12.4))
      PRINT900
  900 FORMAT('0',10X,'WAKE MODE STRENGTHS')
      DO 14 J=1,NMS
   14 PRINT700,(CWJ(J,N),N=1,NSD)
  700 FORMAT('0',10X,10E12.4)
      CALL STEADY(AR,AL,TH,NMC,NCP,CP,YH,RHS,RSL,CO,DWS,NMS)
      PRINT850
  850 FORMAT('0',10X,'STEADY STATE MODE STRENGTHS')
      DO 18 I=1,NMC
```

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```
 18 PRINT700,(I,J,CO((I-1)*NMS+J),J=1,NMS)
    CALL TAUS(NMC,NMS,NSF,CCU,CO,H,TAU,A)
    PRINT550
550 FORMAT('0',10X,'EXPONENTIAL APPROXIMATIONS TO MODE HISTORIES')
    DO 16 I=1,NMC
    DO 16 J=1,NMS
    K=NMS*(I-1)+J
    A1=1.-A(K)
 16 PRINT750,I,J,CO(K),A1,TAU(K,1),A(K),TAU(K,2)
750 FORMAT('0',10X,'B(',I1,'-',I1,')=',E12.4,'(1.- ',F12.4,'EXP(',E12.
   1+,'*S)- ',E12.4,'EXP(',E12.4,'*S))')
    CALL FM(NMC,NMS,NSF,RSL,RHS,AR,CCU,CC,CWJ,H,USF,USM,USR,TSF,TYM,TR
   1M)
    PRINT910
910 FORMAT('0',10X,'FORCES & MOMENTS OF MODES')
    PRINT920,(USF(1,J),J=1,NSDS)
    PRINT930
920 FORMAT('0',10X,10E12.4)
930 FORMAT('0')
    PRINT920,(USM(1,J),J=1,NSDS)
    PRINT930
    PRINT920,(USR(1,J),J=1,NSDS)
    DO 22 I=1,NMS
    PRINT930
 22 PRINT920,(USF(I+NMS,J),J=1,NSDS)
    PRINT940
940 FORMAT('0',10X,'TOTAL SIDE FORCE')
    PRINT920,(TSF(J),J=1,NSDS)
    PRINT950
950 FORMAT('0',10X,'TOTAL YAW MOMENT')
    PRINT920,(TYM(J),J=1,NSDS)
    PRINT960
960 FORMAT('0',10X,'TOTAL ROLL MOMENT')
    PRINT920,(TRM(J),J=1,NSDS)
C      CALCULATE STARTING FORCES & MOMENTS
    SSF=-1.5708*B1(1)*SF(1)*2.
    SYM=SSF*.5
    SRM=RM(1)*SSF/SF(1)
    DO 24 I=2,NMS
    SSFA=-1.5708*B1(I)*SF(I)*2.
    SSF=SSF+SSFA
    SYM=SYM+.5*SSFA
 24 SRM=SRM+RM(I)*SSFA/SF(I)
    SRM=SRM+SSF*(RSL-AR)
    PRINT650
650 FORMAT('0',10X,'STARTING FORCES & MOMEMTS')
    PRINT675,SSF,SYM,SRM
675 FORMAT('0',2X,'SF=',E12.4,2X,'YM=',E12.4,2X,'RM=',E12.4)
C      CALCULATE STEADY FORCES & MOMENTS
    ESF=3.1416*CO(1)*SF(1)*2.
```

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```
      EYM=.5*ESF
      ERM=ESF*RM(1)/SF(1)
      DO 26 I=2,NMS
      ESFA=3.1416*CO(I)*SF(I)*2.
      ESF=ESF+ESFA
      EYM=EYM+.5*ESFA
   26 ERM=ERM+ESFA*RM(I)/SF(I)
      ERM=ERM+ESF*(RSL-AR)
      DO 27 I=1,NMS
   27 EYM=EYM+.7854*2.*SF(I)*CO(NMS+I)
      PRINT660
  660 FORMAT('0',10X,'STEADY STATE FORCES & MOMENTS')
      PRINT675,ESF,EYM,ERM
   50 FORMAT('1')
      STOP
      END
```

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```
      FUNCTION SINT(X,Y,N,M,AR,YM)
      DIMENSION R(6),E(6),W(3),F(5),C1(6),C2(6)
      Z1(A,B)=B*(1.+A)-1.
      Z2(A,B)=B*(1.-A)+A
      Z3(A,B)=A*(1.+B)+B
      Z4(A,B)=B*(1.-A)+1.
      ZETA=.6366197*ARSIN(Y/AR)
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      IF(ZETA.LT.0.)GO TO 3
      D1=ZETA
      D2=1.-ZETA
      DO 1 I=1,6
      A1=R(I)
      E(I)=Z1(A1,ZETA)
      F(I)=Z2(A1,ZETA)
      B1=E(I)
      B2=F(I)
      ETA1=AR*SIN(1.5708*B1)-Y
      ETA2=AR*SIN(1.5708*B2)-Y
      C1(I)=CINT(X,ETA1,M)
    1 C2(I)=CINT(X,ETA2,M)
      GO TO 5
    3 D1=1.+ZETA
      D2=-1.*ZETA
      DO 4 I=1,6
      A1=R(I)
      E(I)=Z3(A1,ZETA)
      F(I)=Z4(A1,ZETA)
      B1=E(I)
      B2=F(I)
      ETA1=AR*SIN(1.5708*B1)-Y
      ETA2=AR*SIN(1.5708*B2)-Y
      C1(I)=CINT(X,ETA1,M)
    4 C2(I)=CINT(X,ETA2,M)
    5 G1=0.
      G2=0.
      DO 2 II=1,3
      JJ=2*II
      IJ=2*II-1
      X1=E(IJ)
      X2=E(JJ)
      Y1=F(IJ)
```

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```
      Y2=F(JJ)
      W1=C1(IJ)
      W2=C1(JJ)
      R1=C2(IJ)
      R2=C2(JJ)
      G1=G1+W(II)*(F1(X1,Y,W1,AR,N,YH)+F1(X2,Y,W2,AR,N,YH))
    2 G2=G2+W(II)*(F1(Y1,Y,R1,AR,N,YH)+F1(Y2,Y,R2,AR,N,YH))
      SINT=(G1*D1+G2*D2)*.19635
      RETURN
      END
```

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```
      FUNCTION CINT(X,ETA,M)
      DIMENSION R(6),E(6),F(6),W(3)
      Z1(A,B)=A*(1.+B)+B
      Z2(A,B)=B*(1.-A)+1.
      Z3(A,B)=B*(1.+A)-1.
      Z4(A,B)=B*(1.-A)+A
      DEL=.6366197*ARSIN(X)
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      IF(DEL.GT.0.)GO TO 10
      DO 1 I=1,6
      Y=R(I)
      E(I)=Z1(Y,DEL)
    1 F(I)=Z2(Y,DEL)
      A1=1.+DEL
      A2=-1.*DEL
      GO TO 20
   10 DO 2 I=1,6
      Y=R(I)
      E(I)=Z3(Y,DEL)
    2 F(I)=Z4(Y,DEL)
      A1=DEL
      A2=1.-DEL
   20 CINT=0.
      DO 3 I=1,3
      X1=E(2*I-1)
      X2=E(2*I)
      Y1=F(2*I-1)
      Y2=F(2*I)
      D=A1*(FC(X1,ETA,X,M)+FC(X2,ETA,X,M))
      C=A2*(FC(Y1,ETA,X,M)+FC(Y2,ETA,X,M))
    3 CINT=CINT+W(I)*(D+C)
      RETURN
      END
```

```
      FUNCTION FC(X,ETA,XO,M)
      IF(M.EQ.0)G=SIN(1.5708*X)
      IF(M.EQ.1)G=1.-SIN(1.5708*X)
      IF(M.EQ.2)G=-.5*(COS(3.1416*X)+SIN(1.5708*X))
      IF(M.GT.2)G=.5*(COS(M*1.5708*(1.+X))+COS((M-2)*1.5708*(1.+X
      A=SIN(1.5708*X)
      FC=G*SQRT((A-XO)*(A-XO)+ETA*ETA)/(A-XO)
      RETURN
      END
```

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```
      FUNCTION F1(A,Y,P,AP,N,YH)
      IF(N.EQ.1)GO TO 1
      IF(N.EQ.2)GO TO 2
      IF(N.EQ.3)GO TO 3
    1 CONTINUE
      IF(A.LT.YH)GS=COS(1.5708*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=COS(1.5708*(A-1.)/(YH-1.))/(YH-1.)
      GO TO 4
    2 CONTINUE
      IF(A.LT.YH)GS=COS(1.5708*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=COS(1.5708*A/YH)/YH
      IF(A.GE.0.)GS=-2.*COS(A*3.1416)
      GO TO 4
    3 CONTINUE
      IF(A.LT.YH)GS=3.*COS(4.7124*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=3.*COS(4.7124*(A-1.)/(YH-1.))/(YH-1.)
    4 F1=P*GS/(AP*SIN(A*1.5708)-Y)
      RETURN
      END
```

```
      FUNCTION WAKE(H,AR,N,XO,YO,II,TH,YH)
      DIMENSION R(6),E(6),F(6),W(3)
      Z1(A,B)=B*(1.+A)-1.
      Z2(A,B)=B*(1.-A)+A
      Z3(A,B)=A*(1.+B)+B
      Z4(A,B)=B*(1.-A)+1.
      ZETA=.6366197*ARSIN(YO/AR)
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      IF(ZETA.LT.0.)GO TO 4
      D1=ZETA
      D2=1.-ZETA
      DO 2 I=1,6
      A1=R(I)
      E(I)=Z1(A1,ZETA)
    2 F(I)=Z2(A1,ZETA)
      GO TO 6
    4 D1=1.+ZETA
      D2=-1.*ZETA
      DO 5 I=1,6
      A1=R(I)
      E(I)=Z3(A1,ZETA)
    5 F(I)=Z4(A1,ZETA)
    6 WAKE=0.
      DO 3 I=1,3
      R1=E(2*I-1)
      R2=E(2*I)
      E1=F(2*I-1)
      E2=F(2*I)
      G1=D1*(FWA(R1,AR,H,XO,YO,N,II,TH,YH)+FWA(R2,AR,H,XO,YO,N,II
     1)
      G2=D2*(FWA(E1,AR,H,XO,YO,N,II,TH,YH)+FWA(E2,AR,H,XO,YO,N,II
     1)
    3 WAKE=WAKE+W(I)*(G1+G2)
      RETURN
      END
```

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```
      FUNCTION FWA(Y,AP,H,XO,YO,N,II,T,YH)
      GS=FSPAN(Y,II,YH)
      XN1=N*H-(XO-1.)*COS(T)
      XN2=XN1+H
      XN3=XN1-H
      ZO=ABS(XO-1.)*SIN(T)
      Y1=AP*SIN(1.5708*Y)-YO
      Y2=ABS(Y1)
      S=Y1/Y2
      RAB=Y2/ZO
      SR1=SQRT(XN1*XN1+Y1*Y1+ZO*ZO)
      SR2=SQRT(XN2*XN2+Y1*Y1+ZO*ZO)
      SR3=SQRT(XN3*XN3+Y1*Y1+ZO*ZO)
      S1=SQRT(XN1*XN1+ZO*ZO)
      S2=SQRT(XN2*XN2+ZO*ZO)
      S3=SQRT(XN3*XN3+ZO*ZO)
      A=Y1/(Y1*Y1+ZO*ZO)
      G1=A*(.5*(XN2*SR2+XN3*SR3)-XN1*SR1)
      G2=Y1*(ALOG(XN1+SR1)-.5*(ALOG(XN2+SR2)+ALOG(XN3+SR3)))
      G3=S*(XN2*ALOG((SR2-Y2)/S2)+XN3*ALOG((SR3-Y2)/S3)-2.*XN1*AL
     1-Y2)/S1))
      G4=ZO*S*(ATAN(RAB*XN2/SR2)+ATAN(RAB*XN3/SR3)-2.*ATAN(RAB*XN
      FWA=(G1+G2+G3+G4)*GS*.125/H
      FWA=FWA*COS(T)
      RETURN
      END
```

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```
      FUNCTION TECMD(AR,H,N,XO,YO,II,T,YH)
      DIMENSION R(6),W(3),E(6),F(6)
      Z1(A,B)=B*(1.+A)-1.
      Z2(A,B)=B*(1.-A)+A
      Z3(A,B)=A*(1.+B)+B
      Z4(A,B)=B*(1.-A)+1.
      ZETA=.6366197*ARSIN(YO/AR)
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      IF(ZETA.LT.0.)GO TO 4
      D1=ZETA
      D2=1.-ZETA
      DO 2 I=1,6
      A1=R(I)
      E(I)=Z1(A1,ZETA)
    2 F(I)=Z2(A1,ZETA)
      GO TO 6
    4 D1=1.+ZETA
      D2=-1.*ZETA
      DO 5 I=1,6
      A1=R(I)
      E(I)=Z3(A1,ZETA)
    5 F(I)=Z4(A1,ZETA)
    6 TECMD=0.
      DO 3 I=1,3
      Y1=E(2*I-1)
      Y2=E(2*I)
      E1=F(2*I-1)
      E2=F(2*I)
      G1=D1*(FT(Y1,AR,H,XO,YO,N,II,T,YH)+FT(Y2,AR,H,XO,YO,N,II,T
      G2=D2*(FT(E1,AR,H,XO,YO,N,II,T,YH)+FT(E2,AR,H,XO,YO,N,II
    3 TECMD=TECMD+W(I)*(G1+G2)
      RETURN
      END
```

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```
      FUNCTION FT(Y,AR,H,X0,Y0,N,II,T,YH)
      GS=FSPAN(Y,II,YH)
      XN1=N*H-(X0-1.)*COS(T)
      XN2=XN1-H
      XN3=XN2-H
      Y1=AR*SIN(1.5708*Y)-Y0
      Y2=ABS(Y1)
      S=Y1/Y2
      Z0=ABS(X0-1.)*SIN(T)
      RAB=Y2/Z0
      SR1=SQRT(XN1*XN1+Y1*Y1+Z0*Z0)
      SR2=SQRT(XN2*XN2+Y1*Y1+Z0*Z0)
      S1=SQRT(XN1*XN1+Z0*Z0)
      S2=SQRT(XN2*XN2+Z0*Z0)
      A=Y1/(Y1*Y1+Z0*Z0)
      G1=.0625*(1./(XN1*XN1+Z0*Z0)+1./(Y1*Y1+Z0*Z0))*XN1*Y1/SR1
      G2=A*(XN2*SR2-XN3*SR1)
      G3=Y1*ALOG((XN1+SR1)/(XN2+SR2))
      G4=2.*XN2*S*ALOG(S1*(SR2-Y2)/(S2*(SR1-Y2)))
      G5=Z0*S*(ATAN(RAB*XN2/SR2)-ATAN(RAB*XN1/SR1))*2.
      FT=1.5708*(G1+(.125/(H*H))*(G2+G3+G4+G5))*GS/(1.-.5*H)
      FT=FT*COS(T)
      RETURN
      END
```

```
      SUBROUTINE STEADY(AR,AL,TH,NMC,NCP,CP,YH,RHS,RSL,CO,DWS,NMS)
      DIMENSION CP(20,2),CO(12),DWS(20,13),IL(20)
      DIMENSION S(20,13)
      NMDS=NMC*NMS
      DO 1 I=1,NCP
      XO=CP(I,1)
      YO=CP(I,2)
      DWS(I,NMDS+1)=AL*(1.+(RHS/(RSL-AR+YO))**2)
      DO 1 J=1,NMS
    1 DWS(I,J)=SINT(XO,YO,J,1,AR,YH)-TRL(AR,YH,J,XO,YO,TH)
      N1=NMDS+1
      N2=NMDS+2
      DO 2 I=1,NCP
      DO 2 J=1,N1
    2 S(I,J)=DWS(I,J)
      CALL GLSQ(DWS,CO,IL,NCP,NMDS,BUG,0.,0.)
      PRINT50
      DO 3 I=1,NCP
      S(I,N2)=S(I,1)*CO(1)
      DO 4 J=2,NMDS
    4 S(I,N2)=S(I,N2)+S(I,J)*CO(J)
    3 PRINT500,(S(I,J),J=1,N2)
   50 FORMAT('1')
  500 FORMAT('0',10X,10E12.4)
      RETURN
      END
```

```
      FUNCTION TRL(AR,YH,J,X0,Y0,TH)
      DIMENSION R(6),E(6),F(6),W(3)
      Z1(A,B)=A*(1.+B)+B
      Z2(A,B)=B*(1.-A)+1.
      Z3(A,B)=B*(1.+A)-1.
      Z4(A,B)=B*(1.-A)+A
      DEL=.6366197*ARSIN(Y0/AR)
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      IF(DEL.GT.0.)GO TO 10
      DO 1 I=1,6
      Y=R(I)
      E(I)=Z1(Y,DEL)
    1 F(I)=Z2(Y,DEL)
      A1=1.+DEL
      A2=-1.*DEL
      GO TO 20
   10 DO 2 I=1,6
      Y=R(I)
      E(I)=Z3(Y,DEL)
    2 F(I)=Z4(Y,DEL)
      A1=DEL
      A2=1.-DEL
   20 TRL=0.
      DO 3 I=1,3
      X1=E(2*I-1)
      X2=E(2*I)
      Y1=F(2*I-1)
      Y2=F(2*I)
      D=A1*(FTR(AR,X0,Y0,YH,J,TH,X1)+FTR(AR,X0,Y0,YH,J,TH,X2))
      C=A2*(FTR(AR,X0,Y0,YH,J,TH,Y1)+FTR(AR,X0,Y0,YH,J,TH,Y2))
    3 TRL=TRL+W(I)*(C+D)
      RETURN
      END
```

```
      FUNCTION FTR(AR,XO,YO,YH,J,TH,X)
      GS=FSPAN(X,J,YH)
      ETA=AR*SIN(1.5708*X)-YO
      ETA2=ETA*ETA
      Z02=(1.-XO)*(1.-XO)*SIN(TH)*SIN(TH)
      FTR=.3927*GS*ETA*COS(TH)/(ETA2+Z02)
      RETURN
      END
```

```
      FUNCTION ESPAN(A,N,YH)
      IF(N.EQ.1)GO TO 1
      IF(N.EQ.2)GO TO 2
      IF(N.EQ.3)GO TO 3
    1 CONTINUE
      IF(A.LT.YH)GS=COS(1.5708*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=COS(1.5708*(A-1.)/(YH-1.))/(YH-1.)
      GO TO 4
    2 CONTINUE
      IF(A.LT.YH)GS=COS(1.5708*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=COS(1.5708*A/YH)/YH
      IF(A.GE.1.)GS=-2.*COS(A*3.1416)
      GO TO 4
    3 CONTINUE
      IF(A.LT.YH)GS=3.*COS(4.7124*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=3.*COS(4.7124*(A-1.)/(YH-1.))/(YH-1.)
    4 ESPAN=GS
      RETURN
      END
```

```
      SUBROUTINE TAUS(NMC,NMS,NSD,COU,CO,H,TAU,A)
      DIMENSION IL(20)
      DIMENSION COU(9,40),CO(12),TAU(12,2),RAT(20,13),ANS(12),A(12)
      REAL MU1,MU2
      DO 20 I=1,12
      A(I)=0.
      TAU(I,1)=0.
   20 TAU(I,2)=0.
C        FIRST MODE CHORDWISE AND SPANWISE
      N=NSD-1
      RAT(1,1)=1.-COU(1,1)/CO(1)
      RAT(1,2)=1.
      RAT(1,3)=1.-COU(1,2)/CO(1)
      DO 1 I=2,N
      RAT(I,1)=1.-COU(1,I)/CO(1)
      RAT(I,2)=1.-COU(1,I-1)/CO(1)
    1 RAT(I,3)=1.-COU(1,I+1)/CO(1)
      CALL GLSQ(RAT,ANS,IL,N,2,BUG,0.,0.)
      A1=.5*ANS(1)
      SQ=SQRT(A1*A1+ANS(2))
      MU1=A1+SQ
      MU2=A1-SQ
      TAU(1,1)=ALOG(MU1)/H
      TAU(1,2)=ALOG(MU2)/H
      DO 2 I=1,NSD
    2 A(1)=A(1)+(1.-COU(1,I)/CO(1)-MU1**I)/(MU2**I-MU1**I)
      A(1)=A(1)/NSD
C        SECOND MODE CHORDWISE
      DO 5 II=1,NMS
      J=NMS+II
      RAT(1,1)=1.-.5*COU(J,2)/CO(J)
      RAT(1,2)=1.
      RAT(1,3)=1.-COU(J,2)/CO(J)
      DO 3 I=2,N
      RAT(I,1)=1.-COU(J,I)/CO(J)
      RAT(I,2)=1.-COU(J,I-1)/CO(J)
    3 RAT(I,3)=1.-COU(J,I+1)/CO(J)
      RAT(2,2)=RAT(1,1)
      CALL GLSQ(RAT,ANS,IL,N,2,BUG,0.,0.)
      A1=.5*ANS(1)
      SQ=SQRT(A1*A1+ANS(2))
      MU1=A1+SQ
      MU2=A1-SQ
      TAU(J,1)=ALOG(MU1)/H
      TAU(J,2)=ALOG(MU2)/H
      A(J)=(1.-.5*COU(J,2)/CO(J)-MU1)/(MU2-MU1)
      DO 4 I=2,NSD
    4 A(J)=A(J)+(1.-COU(J,I)/CO(J)-MU1**I)/(MU2**I-MU1**I)
    5 A(J)=A(J)/NSD
      RETURN
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      SUBROUTINE ...(CUM,...,NSD,RSL,HC,AR,COU,CC,CWJ,H,USF,USM,USR,TSF,
     1TYM,TRM)
      DIMENSION C (12),USF(12,20),USM(12,20),USR(12,20),TSF(20),TYM(20),
     1TRM(20),SF(5),RM(5),COU(9,40),CWJ(3,40)
      YH=1.-(...-...S)/...
      YHT=.636...7*ARSIN(YH)
      CALL S...(...,YHT,NMS,SF,RM)
      SQ=SQRT(H/2.)
      H1=1.-H/2.
      H2=1.+H/2.
      ZUFC=SQ (1.5708+ARSIN(H1/H2)+H1*2.*SQ/H2**2)/H1-2.*H/H1
      ZSMC=ZUFC-2.*H/H1
      ZUMC=-.5*ZUFC*H2**2+1.333*H*H/H1+.5*H/H1
C     FIRST MODE CHORDWISE
      USF(1,NSD)=2.*SF(1)*3.1416*(COU(1,NSD)+1.5*(COU(1,NSD)-COU(1,NSD-
     11))/H)
      USM(1,NSD)=2.*SF(1)*1.5708*(COU(1,NSD)-.5*(COU(1,NSD)-COU(1,NSD-1)
     1)/H)
      USF(1,1)=2.*SF(1)*(3.1416*COU(1,1)+1.5*CWJ(1,1))
      USM(1,1)=2.*SF(1)*(1.5708*COU(1,1)-.25*CWJ(1,1))
      USF(1,1)=USF(1,1)+2.*SF(1)*ZUFC*(CWJ(1,2)-CWJ(1,1))/H
      USM(1,1)=USM(1,1)+2.*SF(1)*(ZSMC*CWJ(1,1)+ZUMC*(CWJ(1,2)-CWJ(1,1))
     1/H)
      USR(1,1)=USF(1,1)*(RSL-AR+RM(1)/SF(1))
      N1=NSD-1
      DO 1 J=2,N1
      USF(1,J)=2.*SF(1)*3.1416*(COU(1,J)+1.5*(COU(1,J+1)-COU(1,J-1))/(2.
     1*H))
    1 USM(1,J)=2.*SF(1)*1.5708*(COU(1,J)-.5*(COU(1,J+1)-COU(1,J-1))/(2.*
     1H))
C     ADD ... MODE EFFECT
      USF(1,NSD)=USF(1,NSD)+2.*SF(1)*ZUFC*(CWJ(1,NSD)-CWJ(1,NSD-1))/H
      USM(1,NSD)=USF(1,NSD)+2.*SF(1)*(ZSMC*CWJ(1,NSD)+ZUMC*(CWJ(1,NSD)-C
     1WJ(1,NSD-1))/H)
      USR(1,NSD)=USF(1,NSD)*(RSL-AR+RM(1)/SF(1))
      N1=NSD-1
      DO 2 J=2,N1
      USF(1,J)=USF(1,J)+2.*SF(1)*ZUFC*(CWJ(1,J+1)-CWJ(1,J-1))/(2.*H)
      USM(1,J)=USM(1,J)+2.*SF(1)*(ZSMC*CWJ(1,J)+ZUMC*(CWJ(1,J+1)-CWJ(1,J
     1-1))/(2.*H))
    2 USR(1,J)=USF(1,J)*(RSL-AR+RM(1)/SF(1))
C     S... ... CHORDWISE
      DO 3 I=1,...
      I1=I+NMS
      USF(I1,1)=2.*SF(I)*.7854*COU(I1,2)/(2.*H)
      USM(I1,1)=.5*USF(I1,1)+.7854*COU(I1,2)*.5*2.*SF(I)
      USR(I1,1)=USF(I1,1)*(RSL-AR+RM(I)/SF(I))
      USF(I1,NSD)=2.*SF(I)*.7854*(COU(I1,NSD)-COU(I1,NSD-1))/H
      USM(I1,NSD)=.5*USF(I1,NSD)+.7854*COU(I1,NSD)*2.*SF(I)
      USR(I1,NSD)=USF(I1,NSD)*(RSL-AR+RM(I)/SF(I))
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      USF(I1,2)=USF(I)*.7854*(CU(I1,3)-.5*CU(I1,2))/(2.*H)
      USM(I1,2)=.5*USF(I1,2)+.7354*CU(I1,2)*2.*SF(I)
      USF(I1,2)=USF(I1,2)*(FSL-AF+RM(I)/SF(I))
      DO 3 J=2,N1
      USF(I1,J)=2.*SF(I)*.7854*(CU(I1,J+1)-CU(I1,J-1))/(2.*H)
      USM(I1,J)=.5*USF(I1,J)+.7854*CU(I1,J)*2.*SF(I)
    3 USF(I1,J)=USF(I1,J)*(FSL-AF+RM(I)/SF(I))
      DO 4 J=1,NSP
      TSF(J)=USF(1,J)
      TYM(J)=USM(1,J)
      TRM(J)=USF(1,J)
      DO 4 I=1,NS
      I1=I+NS
      TSF(J)=TSF(J)+USF(I1,J)
      TYM(J)=TYM(J)+USM(I1,J)
    4 TRM(J)=TRM(J)+USF(I1,J)
      RETURN
      END
```

```
      SUBROUTINE FS(AR,YH,NMS,SF,RM)
      DIMENSION SF(5),RM(5)
      HI=.2
      DO 1 I=1,NMS
      RP2=-1.+HI
      ANF=0.+4.*FSR(RP2,YH,I,1)+0.
      ANM=0.+4.*FSR(RP2,YH,I,2)+0.
      DO 2 J=3,9,2
      RP1=-1.+J*HI
      RP2=-1.+(J+1)*HI
      ANF=ANF+2.*FSR(RP1,YH,I,1)+4.*FSR(RP2,YH,I,1)
    2 ANM=ANM+2.*FSR(RP1,YH,I,2)+4.*FSR(RP2,YH,I,2)
      SF(I)=AR*HI*ANF*.5236
    1 RM(I)=AR*HI*ANM*.2618
      RETURN
      END
```

[illegible] = 7414[illegible] 21/55/53

```
      FUNCTION [illegible](Y,YH,II,IJ)
      IF(Y.LT.YH)GO TO 3
      IF(II.EQ.2)GO TO 1
      IF(II.EQ.1)GS=SIN(1.5706*(Y-1.)/(YH-1.))
      IF(II.EQ.3)GS=SIN(4.7124*(Y-1.)/(YH-1.))
      GO TO 2
    1 IF(Y.LT.YH)GS=SIN(1.5703*Y/YH)
      IF(Y.GT.0.)GS=-SIN(Y*3.1416)
      GO TO 2
    3 CONTINUE
      GS=SIN(1.5708*(Y+1.)/(YH+1.))
      IF(II.EQ.3)GS=SIN(4.7124*(Y+1.)/(YH+1.))
    2 CONTINUE
      IF(IJ.EQ.1)FSR=GS*COS(1.5708*Y)
      IF(IJ.EQ.2)FSR=GS*SIN(3.1416*Y)
      RETURN
      END
```

```
      SUBROUTINE GLSQ(A,X,IL,N,M,ALPHA,E1,E2)
      DIMENSION A(20,13),X(12),IL(20)
      MM=M+1
      LL=1
      DO 50 J=1,MM
   50 IL(J)=0
      I=1
      DO 3 K=1,MM
      II=I+1
      DO 4 J=II,N
      IF(ABS(A(J,K))-E1)4,4,6
    6 T1=SQRT((A(J,K))**2+(A(I,K))**2)
      S=A(J,K)/T1
      C=A(I,K)/T1
      DO 5 L=K,MM
      T2=C*A(I,L)+S*A(J,L)
      A(J,L)=-S*A(I,L)+C*A(J,L)
    5 A(I,L)=T2
      LL=LL+1
    4 CONTINUE
      IF(ABS(A(I,K))-E2)3,3,8
    8 IL(K)=I
      I=I+1
    3 CONTINUE
      X(MM)=-1.0
      II=M
      DO 35 I=1,M
   35 X(I)=0.0
      DO 30 J=1,M
      IF(IL(II))30,30,31
   31 S=0.0
      LL=II+1
      I=IL(II)
      DO 32 K=LL,MM
   32 S=S+A(I,K)*X(K)
      X(II)=-S/A(I,II)
   30 II=II-1
      IF(IL(MM))50,51,50
   51 ALPHA=0.0
      GO TO 52
   50 I=IL(MM)
      ALPHA=A(I,MM)
   52 RETURN
      END
```

SAIL-RUDDER INTERACTION RESPONSE PROGRAM

INPUT:
1) Vehicle Geometry
2) Trailing Vortex Sheet Angle
3) Exponential Approximations to Circulation Response of Sail

OUTPUT:
1) Step by Step Forces and Moments on Rudder due to Unsteady Sail Wake

```
      DIMENSION X(40),RT(40),CP(20,2),RVP(20,2),DWI(40,40,3)
      DIMENSION DWB(40,40,3),DWR1(20,13),DWR(20,13),B(20),COL(12,40)
      DIMENSION SF(5),RM(5)
      DIMENSION RUF(40),RUM(40),RUR(40),FM(3,40),CFM(3,40),DFM(3,40)
      DIMENSION IL(20)
      READ100,NMC,NMS,NCPC,NCPS,NSD,NX,TH,RHR,RRD,RL,HCR,RHS,RSL,BOW,STE
     1RN
  100 FORMAT(6I5,9F5.3)
      READ110,FVS,A,T1,T2
  110 FORMAT(4E12.4)
      NPTS=NCPC*NCPS
      NMDS=NMS*NMC
      READ200,(CP(I,1),I=1,NPTS)
  200 FORMAT(16F5.3)
      ARR=.5*(RRD-RHR)*(RRD+RHR)/RRD/HCR
      Y1=1.-(RRD-RHR)/ARR/HCR
      ARS=.5*(RSL-RHS)*(RSL+RHS)/RSL
      YH=.6366197*ARSIN(Y1)
      DO 1 I=1,NCPC
    1 CP(1+(I-1)*NCPS,2)=(Y1+.1)*ARR
      DO 2 I=1,NCPC
      DO 2 J=2,NCPS
    2 CP(J+(I-1)*NCPS,2)=(2*J-3)*.5*ARR/(NCPS-1)
      ROR=RRD-ARR*HCR
      ROS=RSL-ARS
      DO 3 I=1,NPTS
      RVP(I,1)=CP(I,1)*HCR+RL
    3 RVP(I,2)=CP(I,2)*HCR+ROR-ROS
      X(1)=1.
      RT(1)=RSL
      CALL WCT(BOW,STERN,X,RT,NSD)
  925 FORMAT('0',2X,E12.4,2X,E12.4)
      DO 4 I=1,NPTS
      X0=RVP(I,1)
      Y0=RVP(I,2)
      CALL UTRLD(X0,Y0,TH,X,ARS,RT,1,NX,NSD,I,DWI,RHS,RSL)
    4 CALL UPND(X0,Y0,TH.X,RT,ARS,1,NX,NSD,I,DWB,RHS,RSL)
      DO 5 I=1,NPTS
      PRINT300,(DWI(I,J,1),J=1,NX)
    5 PRINT300,(DWB(I,J,1),J=1,NX)
      DO 6 I=1,NPTS
      X0=CP(I,1)
      Y0=CP(I,2)
      DO 6 IJ=1,NMC
      DO 6 J=1,NMS
      DWR1(I,(IJ-1)*NMS+J)=SINT(X0,Y0,J,IJ,ARR,YH)
      IF(IJ.EQ.1)DWR1(I,J)=DWR1(I,J)+TRL(ARR,YH,J,X0,Y0,0.)
    6 CONTINUE
      PRINT 900
  900 FORMAT('0',2X,'STATEMENT 1')
```

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```
              PRINT925,(X(I),RT(I),I=1,NSD)
              DO 7 I=1,NPTS
              DWR1(I,NMDS+1)=DWB(I,1,1)
              NE=NMDS+1
              DO 7 J=1,NE
            7 DWR(I,J)=DWR1(I,J)
              CALL GLSQ(DWR,B,IL,NPTS,NMDS,BUG,0.,0.)
              DO 8 I=1,NMDS
            8 COU(I,1)=B(I)
              DO 9 I=2,NX
              DO 10 J=1,NPTS
              DWR1(J,NMDS+1)=DWR1(J,NMDS+1)+DWB(J,I,1)-DWB(J,I-1,1)+DWI(J,1,1)
              DWR(J,NMDS+1)=DWR1(J,NMDS+1)
              DO 10 IJ=1,NMDS
           10 DWR(J,IJ)=DWR1(J,IJ)
              PRINT400,I
          400 FORMAT('0',5X,'DOWNWASHES AT STEP',I3)
              PRINT300,(DWR(IK,NMDS+1),IK=1,NPTS)
              CALL GLSQ(DWR,B,IL,NPTS,NMDS,BUG,0.,0.)
              DO 9 IJ=1,NMDS
            9 COU(IJ,I)=B(IJ)
              PRINT950
          950 FORMAT('0',2X,'STATEMENT 2')
              PRINT925,(X(I),RT(I),I=1,NX)
              PRINT350
          350 FORMAT('0',2X,'CIRCULATION RESPONSE OF RUDDER TO UNIT STEP')
              DO 11 I=1,NMDS
              PRINT375,I
          375 FORMAT('0',2X,'MODE',I2)
           11 PRINT300,(COU(I,J),J=1,NX)
          300 FORMAT('0',2X,10E12.4)
C               CALCULATE FORCES & MOMENTS
              CALL SR(ARR,YH,NMS,SF,RM)
              DO 12 I=1,NX
              RUF(I)=-COU(1,I)*3.1416*2.*SF(1)*HCR**2
              RUM(I)=2.*SF(1)*1.5708*COU(1,I)*HCR**3
              RUR(I)=-COU(1,I)*3.1416*2.*RM(1)*HCR**3
              DO 13 J=2,NMS
              RUF(I)=RUF(I)-COU(J,I)*3.1416*2.*SF(J)*HCR**2
              RUM(I)=RUM(I)+2.*SF(J)*1.5708*COU(J,I)*HCR**3
           13 RUR(I)=RUR(I)-3.1416*2.*RM(J)*COU(J,I)*HCR**3
C               SECOND MODE CHORDWISE
              DO 12 J=1,NMS
           12 RUM(I)=RUM(I)-.7854*COU(J+NMS,I)*2.*SF(J)*HCR**3
              DO 18 I=1,NX
              RUM(I)=RUM(I)-RUF(I)*RL
           18 RUR(I)=RUR(I)+RUF(I)*(RRD-ARR*HCR)
              PRINT380
          380 FORMAT('0',2X,' FORCE RESPONSE OF RUDDER TO UNIT STEP')
           14 PRINT390,(X(J),RUF(J),RUM(J),RUR(J),J=1,NX)
```

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```
   390 FORMAT('0',2X,'X=',E12.4,2X,'SF=',E12.4,2X,'YM=',E12.4,2X,'RM=',F1
      12.4)
       DO 15 I=1,NX
       FM(1,I)=RUF(I)
       FM(2,I)=RUM(I)
    15 FM(3,I)=RUR(I)
       H=X(6)-X(5)
       DO 16 I=1,3
       CALL DERY(NX,H,FM,DFM,I)
       CALL CONV(NX,H,DFM,FM,A,T1,T2,CFM,I)
       DO 16 J=1,NX
    16 CFM(I,J)=CFM(I,J)*3.1416*FVS
       PRINT395
   395 FORMAT('0',2X,'CONVOLVED FORCE & MOMENT RESPONSE')
       PRINT410,(X(I),CFM(1,I),CFM(2,I),CFM(3,I),I=1,NX)
   410 FORMAT('0',2X,'X=',E12.4,2X,'SF=',E12.4,2X,'YM=',2X,E12.4,2X,'RM='
      1,F12.4)
       STOP
       END
```

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```
      FUNCTION SINT(X,Y,N,M,AR,YH)
      DIMENSION R(6),E(6),W(3),F(6),C1(6),C2(6)
      Z1(A,B)=B*(1.+A)-1.
      Z2(A,B)=B*(1.-A)+A
      Z3(A,B)=A*(1.+B)+B
      Z4(A,B)=B*(1.-A)+1.
      ZETA=.6366197*ARSIN(Y/AR)
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      IF(ZETA.LT.0.)GO TO 3
      D1=ZETA
      D2=1.-ZETA
      DO 1 I=1,6
      A1=R(I)
      E(I)=Z1(A1,ZETA)
      F(I)=Z2(A1,ZETA)
      B1=E(I)
      B2=F(I)
      ETA1=AR*SIN(1.5708*B1)-Y
      ETA2=AR*SIN(1.5708*B2)-Y
      C1(I)=CINT(X,ETA1,M)
    1 C2(I)=CINT(X,ETA2,M)
      GO TO 5
    3 D1=1.+ZETA
      D2=-1.*ZETA
      DO 4 I=1,6
      A1=R(I)
      E(I)=Z3(A1,ZETA)
      F(I)=Z4(A1,ZETA)
      B1=E(I)
      B2=F(I)
      ETA1=AR*SIN(1.5708*B1)-Y
      ETA2=AR*SIN(1.5708*B2)-Y
      C1(I)=CINT(X,ETA1,M)
    4 C2(I)=CINT(X,ETA2,M)
    5 G1=0.
      G2=0.
      DO 2 II=1,3
      JJ=2*II
      IJ=2*II-1
      X1=E(IJ)
      X2=E(JJ)
      Y1=F(IJ)
```

```
      Y2=F(JJ)
      W1=C1(IJ)
      W2=C1(JJ)
      R1=C2(IJ)
      R2=C2(JJ)
      G1=C1+W(II)*(F1(X1,Y,W1,AR,N,YH)+F1(X2,Y,W2,AR,N,YH))
    2 G2=G2+W(II)*(F1(Y1,Y,R1,AR,N,YH)+F1(Y2,Y,R2,AR,N,YH))
      SINT=(G1*D1+G2*D2)*.19635
      RETURN
      END
```

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```
      FUNCTION CINT(X,ETA,M)
      DIMENSION R(6),E(6),F(6),W(3)
      Z1(A,B)=A*(1.+B)+B
      Z2(A,B)=B*(1.-A)+1.
      Z3(A,B)=B*(1.+A)-1.
      Z4(A,B)=B*(1.-A)+A
      DEL=.6366197*ARSIN(X)
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      IF(DEL.GT.0.)GO TO 10
      DO 1 I=1,6
      Y=R(I)
      E(I)=Z1(Y,DEL)
    1 F(I)=Z2(Y,DEL)
      A1=1.+DEL
      A2=-1.*DEL
      GO TO 20
   10 DO 2 I=1,6
      Y=R(I)
      E(I)=Z3(Y,DEL)
    2 F(I)=Z4(Y,DEL)
      A1=DEL
      A2=1.-DEL
   20 CINT=0.
      DO 3 I=1,3
      X1=E(2*I-1)
      X2=E(2*I)
      Y1=F(2*I-1)
      Y2=F(2*I)
      D=A1*(FC(X1,ETA,X,M)+FC(X2,ETA,X,M))
      C=A2*(FC(Y1,ETA,X,M)+FC(Y2,ETA,X,M))
    3 CINT=CINT+W(I)*(D+C)
      RETURN
      END
```

```
      FUNCTION FC(X,ETA,XO,M)
      IF(M.EQ.0)G=SIN(1.5708*X)
      IF(M.EQ.1)G=1.-SIN(1.5708*X)
      IF(M.EQ.2)G=-.5*(COS(3.1416*X)+SIN(1.5708*X))
      IF(M.GT.2)G=.5*(COS(M*1.5708*(1.+X))+COS((M-2)*1.5708*(1.+X)))
      A=SIN(1.5708*X)
      FC=G*SQRT((A-XO)*(A-XO)+ETA*ETA)/(A-XO)
      RETURN
      END
```

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```
      FUNCTION F1(A,Y,B,AR,N,YH)
      IF(N.EQ.1)GO TO 1
      IF(N.EQ.2)GO TO 2
      IF(N.EQ.3)GO TO 3
    1 CONTINUE
      IF(A.LT.YH)GS=COS(1.5708*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=COS(1.5708*(A-1.)/(YH-1.))/(YH-1.)
      GO TO 4
    2 CONTINUE
      IF(A.LT.YH)GS=COS(1.5708*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=COS(1.5708*A/YH)/YH
      IF(A.GE.0.)GS=-2.*COS(A*3.1416)
      GO TO 4
    3 CONTINUE
      IF(A.LT.YH)GS=3.*COS(4.7124*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=3.*COS(4.7124*(A-1.)/(YH-1.))/(YH-1.)
    4 F1=B*GS/(AR*SIN(A*1.5708)-Y)
      RETURN
      END
```

```
      FUNCTION TRL(AR,YH,J,XO,YO,TH)
      DIMENSION R(6),E(6),F(6),W(3)
      Z1(A,B)=A*(1.+B)+B
      Z2(A,B)=B*(1.-A)+1.
      Z3(A,B)=B*(1.+A)-1.
      Z4(A,B)=B*(1.-A)+A
      DEL=.6366197*ARSIN(YO/AR)
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      IF(DEL.GT.C.)GO TO 10
      DO 1 I=1,6
      Y=R(I)
      E(I)=Z1(Y,DEL)
    1 F(I)=Z2(Y,DEL)
      A1=1.+DEL
      A2=-1.*DEL
      GO TO 20
   10 DO 2 I=1,6
      Y=R(I)
      E(I)=Z3(Y,DEL)
    2 F(I)=Z4(Y,DEL)
      A1=DEL
      A2=1.-DEL
   20 TRL=0.
      DO 3 I=1,3
      X1=E(2*I-1)
      X2=E(2*I)
      Y1=F(2*I-1)
      Y2=F(2*I)
      D=A1*(FTR(AR,XO,YO,YH,J,TH,X1)+FTR(AR,XO,YO,YH,J,TH,X2))
      C=A2*(FTR(AR,XO,YO,YH,J,TH,Y1)+FTR(AR,XO,YO,YH,J,TH,Y2))
    3 TRL=TRL+W(I)*(C+D)
      RETURN
      END
```

```
      FUNCTION FTR(AR,XO,YO,YH,J,TH,X)
      GS=FSPAN(X,J,YH)
      ETA=AR*SIN(1.5708*X)-YO
      ETA2=ETA*ETA
      ZO2=(1.-XO)*(1.-XO)*SIN(TH)*SIN(TH)
      FTR=.3927*GS*ETA*COS(TH)/(ETA2+ZO2)
      RETURN
      END
```

```
      FUNCTION FSPAN(A,N,YH)
      IF(N.EQ.1)GO TO 1
      IF(N.EQ.2)GO TO 2
      IF(N.EQ.3)GO TO 3
    1 CONTINUE
      IF(A.LT.YH)GS=COS(1.5708*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=COS(1.5708*(A-1.)/(YH-1.))/(YH-1.)
      GO TO 4
    2 CONTINUE
      IF(A.LT.YH)GS=COS(1.5708*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=COS(1.5708*A/YH)/YH
      IF(A.GE.0.)GS=-2.*COS(A*3.1416)
      GO TO 4
    3 CONTINUE
      IF(A.LT.YH)GS=3.*COS(4.7124*(A+1.)/(YH+1.))/(YH+1.)
      IF(A.GE.YH)GS=3.*COS(4.7124*(A-1.)/(YH-1.))/(YH-1.)
    4 FSPAN=GS
      RETURN
      END
```

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```
          SUBROUTINE GLSQ(A,X,IL,N,M,ALPHA,E1,E2)
          DIMENSION A(20,13),X(20),IL(20)                        GLSQ
          MM=M+1                                                 GLSQ
          LL=1                                                   GLSQ
          DO 60 J=1,MM                                           GLSQ
   60     IL(J)=0                                                GLSQ
          I=1                                                    GLSQ
          DO 3 K=1,MM                                            GLSQ
          II=I+1                                                 GLSQ
          DO 4 J=II,N                                            GLSQ
          IF(ABS(A(J,K))-E1)4,4,6                                GLSQ
    6     T1=SQRT((A(J,K))**2+(A(I,K))**2)                       GLSQ
          S=A(J,K)/T1                                            GLSQ
          C=A(I,K)/T1                                            GLSQ
          DO 5 L=K,MM                                            GLSQ
          T2=C*A(I,L)+S*A(J,L)
          A(J,L)=-S*A(I,L)+C*A(J,L)                              GLSQ
    5     A(I,L)=T2                                              GLSQ
          LL=LL+1                                                GLSQ
    4     CONTINUE                                               GLSQ
          IF(ABS(A(I,K))-E2)3,3,8                                GLSQ
    8     IL(K)=I                                                GLSQ
          I=I+1                                                  GLSQ
    3     CONTINUE                                               GLSQ
          X(MM)=-1.0                                             GLSQ
          II=M                                                   GLSQ
          DO 35 I=1,M                                            GLSQ
   35     X(I)=0.0                                               GLSQ
          DO 30 J=1,M                                            GLSQ
          IF(IL(II))30,30,31                                     GLSQ
   31     S=0.0                                                  GLSQ
          LL=II+1                                                GLSQ
          I=IL(II)                                               GLSQ
          DO 32 K=LL,MM                                          GLSQ
   32     S=S+A(I,K)*X(K)                                        GLSQ
          X(II)=-S/A(I,II)                                       GLSQ
   30     II=II-1                                                GLSQ
          IF(IL(MM))50,51,50                                     GLSQ
   51     ALPHA=0.0                                              GLSQ
          GO TO 52                                               GLSQ
   50     I=IL(MM)                                               GLSQ
          ALPHA=A(I,MM)                                          GLSQ
   52     RETURN                                                 GLSQ
          END
```

```
      SUBROUTINE UBND(XO,YO,T,X,RT,AR,NMS,NX,NSD,ICP,DWB,RHS,RSL)
      DIMENSION X(40),RT(40),DWB(40,40,3),R(6),E(6),F(6),W(3)
      Z1(A,B,C)=.5*(A*(C-B)+C+B)
      H=X(2)-X(1)
      ZO=(XO-1.)*SIN(T)
      XO1=(XO-1.)*COS(T)
      DO 1 I=1,NMS
      DO 1 N=1,NX
      IF(N.GT.NSD)GO TO 6
      XN=X(N)-1.
      RTE=RT(N)
      XH=X(N)
      RH=HRAD(XH)
      ARN=AR*(RTE-RH)/(RSL-RHS)
      YO1=YO-RSL+AR+RTE-ARN
      YH1=ARN-RTE+RH
      YHT=ARSIN(YH1/ARN)*.6366197
    6 CONTINUE
      IF(N.GT.NSD)X(N)=X(N-1)+H
      IF(N.GT.NSD)XN=XN+H
      IF(YO1.GE.YH1)GO TO 2
      HI=.1*(1.-YHT)
      Y1=YHT+HI
      VINT=FUBND(YHT,XO1,YO1,ZO,XN,ARN,YHT,I)+4.*FUBND(Y1,XO1,YO1,ZO,XN,
     1ARN,YHT,I)+FUBND(1.,XO1,YO1,ZO,XN,ARN,YHT,I)
      DO 3 J=2,8,2
      Y1=YHT+J*HI
      Y2=Y1+HI
    3 VINT=VINT+2.*FUBND(Y1,XO1,YO1,ZO,XN,ARN,YHT,I)+4.*FUBND(Y2,XO1,YO1
     1,ZO,XN,ARN,YHT,I)
      DWB(ICP,N,I)=HI*VINT/3.
      GO TO 1
    2 DIF=.6366197*ARSIN(YO1)-YHT
      UP=YHT+2.*DIF
      IF(UP.GT.1.)UP=2.*(DIF+YHT)-1.
      D1=(UP-YHT)*.5
      D2=(1.-UP)*.5
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      DO 4 II=1,6
      A1=R(II)
      E(II)=Z1(A1,YHT,UP)
    4 F(II)=Z1(A1,UP,1.)
```

```
      G1=0.
      G2=0.
      DO 5 II=1,3
      X1=E(2*II-1)
      X2=E(2*II)
      W1=F(2*II-1)
      W2=F(2*II)
      G1=G1+W(II)*(FUBND(X1,X01,Y01,Z0,XN,ARN,YHT,I)+FUBND(X2,X01,Y01,Z0
     1,XN,ARN,YHT,I))
    5 G2=G2+W(II)*(FUBND(W1,X01,Y01,Z0,XN,ARN,YHT,I)+FUBND(W2,X01,Y01,Z0
     1,XN,ARN,YHT,I))
      DWB(ICP,N,I)=(G1*D1+G2*D2)
    1 CONTINUE
      RETURN
      END
```

```
      FUNCTION FURND(Y,XO,YO,ZO,XN,AR,YH,II)
      IF(II.EQ.2)GO TO 1
      IF(II.EQ.1)GS=SIN(1.5708*(Y-1.)/(YH-1.))
      IF(II.EQ.3)GS=SIN(4.7124*(Y-1.)/(YH-1.))
      GO TO 2
    1 IF(Y.GT.YH)GS=SIN(1.5708*Y/YH)
      IF(Y.GE.0.)GS=-SIN(Y*3.1416)
    2 CONTINUE
      ETA=AR*SIN(Y*1.5708)-YO
      DEN=ETA*ETA+ZO*ZO+(XN-XO)*(XN-XO)
      FURND=.125*AR*(XN-XO)*GS*COS(1.5708*Y)/DEN**1.5
      RETURN
      END
```

```
      SUBROUTINE UTRLD(XO,YO,T,X,AR,RT,NMS,NX,NSD,ICP,DWI,RHS,RSL)
      DIMENSION X(40),RT(40),DWI(40,40,3),R(6),E(6),F(6),W(3)
      Z1(A,B,C)=.5*(A*(C-B)+C+B)
      PRINT100,XO,YO,T,AR,NMS,NX,NSD,ICP,RHS,RSL
  100 FORMAT('0',4E12.4,4I5,2F12.4)
      H=X(2)-X(1)
      ZO=(XO-1.)*SIN(T)
      XO1=(XO-1.)*COS(T)
      DO 1 I=1,NMS
      DO 1 N=2,NX
      IF(N.GT.NSD)GO TO 6
      XN=X(N)-1.
      XC=X(N)-.5*H
      XT=XN-H
      RTC=(RT(N)+RT(N-1))*.5
      RHC=HRAD(XC)
      ARN=AR*((RTC-RHC)/(RSL-RHS))
      YO1=YO-RSL+AR+RTC-ARN
      YH1=ARN-RTC+RHC
      YHT=ARSIN(YH1/ARN)*.6366197
    6 CONTINUE
      IF(N.GT.NSD)X(N)=X(N-1)+H
      IF(N.GT.NSD)XN=XN+H
      IF(N.GT.NSD)XT=XT+H
      IF(YO1.GE.YH1)GO TO 2
      HI=.1*(1.-YHT)
      Y1=YHT+HI
      VINT=FUTRLD(YHT,XO1,YO1,ZO,XN,XT,ARN,YHT,I)+4.*FUTRLD(Y1,XO1,YO1,Z
     1O,XN,XT,ARN,YHT,I)+FUTRLD(1.,XO1,YO1,ZO,XN,XT,ARN,YHT,I)
      DO 3 J=2,8,2
      Y1=YHT+J*HI
      Y2=Y1+HI
    3 VINT=VINT+2.*FUTRLD(Y1,XO1,YO1,ZO,XN,XT,ARN,YHT,I)+4.*FUTRLD(Y2,XO
     11,YO1,ZO,XN,XT,ARN,YHT,I)
      DWI(ICP,N,I)=HI*VINT/3.
      GO TO 1
    2 DIF=.6366197*ARSIN(YO1)-YHT
      UP=YHT+2.*DIF
      IF(UP.GT.1.)UP=2.*(DIF+YHT)-1.
      D1=(UP-YHT)*.5
      D2=(1.-UP)*.5
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
```

```
      DO 4 II=1,6
      A1=R(II)
      E(II)=Z1(A1,YHT,UP)
    4 F(II)=Z1(A1,UP,1.)
      G1=0.
      G2=0.
      DO 5 II=1,3
      X1=E(2*II-1)
      X2=E(2*II)
      W1=F(2*II-1)
      W2=F(2*II)
      G1=G1+W(II)*(FUTRLD(X1,XO1,YO1,ZO,XN,XT,ARN,YHT,I)+FUTRLD(X2,XO1,Y
     1O1,ZO,XN,XT,ARN,YHT,I))
    5 G2=G2+W(II)*(FUTRLD(W1,XO1,YO1,ZO,XN,XT,ARN,YHT,I)+FUTRLD(W2,XO1,Y
     1O1,ZO,XN,XT,ARN,YHT,I))
      DWI(ICP,N,I)=(G1*D1+G2*D2)
    1 CONTINUE
      RETURN
      END
```

```
      FUNCTION FUTRLD(Y,XO,YO,ZO,XN,XT,AR,YH,II)
      IF(II.EQ.2)GO TO 1
      IF(II.EQ.1)GS=COS(1.5708*(Y-1.)/(YH-1.))/(YH-1.)
      IF(II.EQ.3)GS=3.*COS(4.7124*(Y-1.)/(YH-1.))/(YH-1.)
      GO TO 2
    1 IF(Y.GT.YH)GS=COS(1.5708*Y/YH)/YH
      IF(Y.GE.0.)GS=-2.*COS(Y*3.1416)
    2 CONTINUE
      ETA=AR*SIN(Y*1.5708)-YO
      DEN=ETA*ETA+ZO*ZO
      F1=(XN-XO)/SQRT((XN-XO)*(XN-XO)+DEN)
      F2=(XT-XO)/SQRT((XT-XO)*(XT-XO)+DEN)
      FUTRLD=-.125*GS*ETA*(F1-F2)/DEN
      RETURN
      END
```

```
      FUNCTION HRAD(X)
      BOW=-5.125
      STERN=7.031
      X1=-4.25
      X2=.875
      X3=6.
      R1=.4351
      R2=.8438
      R3=.3281
      IF(X.LE.X1)GO TO 1
      IF(X.GE.X3)GO TO 2
      A=(R1+R3-2.*R2)*.5/(X2-X1)**2
      B=(R1-R3)/(X1-X3)
      C=R2
      XD=X-X2
      R=A*XD*XD+B*XD+C
      GO TO 3
    1 R=R1*SQRT((X-BOW)/(X1-BOW))
      GO TO 3
    2 R=R3*SQRT((X-STERN)/(X3-STERN))
    3 HRAD=R
      RETURN
      END
```

```
      SUBROUTINE WCT(A,B,X,R,N)
      DIMENSION X(40),R(40)
      FINT(E,F,G,I,D)=E/((F-1.-(I-1)*D)*(F-1.-(I-1)*D)+G*G)**1.5
      H=.025*(B-A)
      HT=(B-1.)/N
      N1=N+1
      DO 1 J=1,N1
      RO=R(J)
      E1=ASLP(A)
      A1=A+H
      E2=ASLP(A1)
      E3=ASLP(B)
      XINT=FINT(E1,A,RO,J,HT)+4.*FINT(E2,A1,RO,J,HT)+FINT(E3,B,RO,J,HT)
      DO 2 IJ=2,38,2
      A1=A+IJ*H
      A2=A1+H
      E1=ASLP(A1)
      E2=ASLP(A2)
    2 XINT=XINT+2.*FINT(E1,A1,RO,J,HT)+4.*FINT(E2,A2,RO,J,HT)
      XINT=H*HT*XINT/18.85
      R(J+1)=R(J)*(1.+XINT)
    1 X(J+1)=X(J)+HT
      RETURN
      END
```

```
      FUNCTION ASLP(X)
      BOW=-5.125
      STERN=7.031
      X1=-4.25
      X2=.875
      X3=6.
      R1=.4351
      R2=.8438
      R3=.3281
      IF(X.LE.X1)GO TO 1
      IF(X.GE.X3)GO TO 2
      A=(R1+R3-2.*R2)*.5/(X2-X1)**2
      B=(R1-R3)/(X1-X3)
      C=R2
      XD=X-X2
      ASLP=6.2832*(2.*A*A*XD*XD*XD+3.*A*B*XD*XD+(B*B+2.*A*C)*XD+C*B)
      GO TO 3
    1 ASLP=3.1416*R1*R1/(X1-BOW)
      GO TO 3
    2 ASLP=3.1416*R3*R3/(X3-STERN)
    3 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DERY(N,H,F,DF,II)
      DIMENSION F(3,40),DF(3,40)
      HI=1./H
      DF(II,N)=HI*(F(II,N)-F(II,N-1))
      DF(II,1)=HI*(F(II,2)-F(II,1))
      N1=N-1
      DO 1 I=2,N1
    1 DF(II,I)=.5*HI*(F(II,I+1)-F(II,I-1))
      RETURN
      END
```

```
      SUBROUTINE SR(AR,YH,NMS,SF,RM)
      DIMENSION SF(5),RM(5)
      HI=.2
      DO 1 I=1,NMS
      BP2=-1.+HI
      ANF=0.+4.*FSR(BP2,YH,I,1)+0.
      ANM=0.+4.*FSR(BP2,YH,I,2)+0.
      DO 2 J=3,9,2
      BP1=-1.+J*HI
      BP2=-1.+(J+1)*HI
      ANF=ANF+2.*FSR(BP1,YH,I,1)+4.*FSR(BP2,YH,I,1)
    2 ANM=ANM+2.*FSR(BP1,YH,I,2)+4.*FSR(BP2,YH,I,2)
      SF(I)=AR*HI*ANF*.5236
    1 RM(I)=AR*HI*ANM*.2618
      RETURN
      END
```

105


```
      FUNCTION FSR(Y,YH,II,IJ)
      IF(Y.LT.YH)GO TO 3
      IF(II.EQ.2)GO TO 1
      IF(II.EQ.1)GS=SIN(1.5708*(Y-1.)/(YH-1.))
      IF(II.EQ.3)GS=SIN(4.7124*(Y-1.)/(YH-1.))
      GO TO 2
    1 IF(Y.GT.YH)GS=SIN(1.5708*Y/YH)
      IF(Y.GE.0.)GS=-SIN(Y*3.1416)
      GO TO 2
    3 CONTINUE
      GS=SIN(1.5708*(Y+1.)/(YH+1.))
      IF(II.EQ.3)GS=SIN(4.712*(Y+1.)/(YH+1.))
    2 CONTINUE
      IF(IJ.EQ.1)FSR=GS*COS(1.5708*Y)
      IF(IJ.EQ.2)FSR=GS*SIN(3.1416*Y)
      RETURN
      END
```

```
      SUBROUTINE CONV(N,H,F,FI,A1,T1,T2,ANS,IJ)
      DIMENSION F(3,40),FI(3,40),ANS(3,40),G(40)
      G(1)=1.
      E1=EXP(T1*H)
      E2=EXP(T2*H)
      IE1=-12/(T1*H)
      IE2=-12/(T2*H)
      DO 1 I=2,N
    1 G(I)=0.
      DO 2 I=2,IE1
    2 G(I)=G(I)+A1*E1**(I-1)
      DO 3 I=2,IE2
      I1=I-1
    3 G(I)=G(I)+(1.-A1)*E2**(I-1)
      ANS(IJ,1)=0.
      DO 4 I=2,N
    4 ANS(IJ,I)=FI(IJ,I)-FI(IJ,1)*G(I)
C        DO FIRST INTEGRABLE STEP
      ANS(IJ,2)=ANS(IJ,2)-.5*H*(F(IJ,1)*G(2)+F(IJ,2)*G(1))
C        DO CASE OF EVEN NUMBER OF BASE POINTS
      DO 5 I=4,N,2
      ANS(IJ,I)=ANS(IJ,I)-.5*H*(F(IJ,1)*G(I)+F(IJ,2)*G(I-1))
      NP=I
      AN1=F(IJ,2)*G(I-1)+4.*F(IJ,3)*G(I-2)+F(IJ,I)*G(1)
      IF(NP.EQ.4)GO TO 5
      NE=NP-2
      DO 6 J=4,NE,2
    6 AN1=AN1+2.*F(IJ,J)*G(I-J+1)+4.*F(IJ,J+1)*G(I-J)
    5 ANS(IJ,I)=ANS(IJ,I)-H*AN1/3.
C        DO CASE OF ODD NUMBER OF BASE POINTS
      DO 8 I=3,N,2
      AN1=F(IJ,1)*G(I)+4.*F(IJ,2)*G(I-1)+F(IJ,I)*G(1)
      IF(I.EQ.3)GO TO 8
      NE=I-2
      DO 9 J=3,NE,2
    9 AN1=AN1+2.*F(IJ,J)*G(I-J+1)+4.*F(IJ,J+1)*G(I-J)
    8 ANS(IJ,I)=ANS(IJ,I)-H*AN1/3.
  200 FORMAT('0',2X,'G(',I2,')=',E12.4)
      RETURN
      END
```

SAIL-HULL INTERACTION RESPONSE PROGRAM

INPUT:
1) Vehicle Geometry
2) Trailing Vortex Sheet Angle
3) Exponential Approximations to Circulation Response of Sail

OUTPUT:
1) Step by Step Forces and Moments on Hull due to Unsteady Sail Wake

```
      DIMENSION VP(40,2),X(40),RT(40),DWB(40,40,3)
      DIMENSION DWI(40,40,3),UF(3,40),UM(3,40)
      DIMENSION DUF(3,40),DUM(3,40),CUF(3,40),CUM(3,40)
      READ100,NVP,NMS,NSD,NX,RHS,RSL,AR,TH,BOW,STERN
      READ200,(VP(I,1),I=1,NVP)
      READ200,(VP(I,2),I=1,NVP)
      READ150,C,T1,T2,A
  150 FORMAT(4E12.4)
      X(1)=1.
      RT(1)=RSL
      CALL WCT(BOW,STERN,X,RT,NSD)
      PRINT400,(X(I),RT(I),I=1,NSD)
  400 FORMAT('0',2X,'X=',E12.4,4X,'R=',E12.4)
      DO 1 I=1,NVP
      X0=VP(I,1)
      Y0=VP(I,2)
      CALL UTRLD(X0,Y0,TH,X,AR,RT,NMS,NX,NSD,I,DWI,RHS,RSL)
    1 CALL UBND(X0,Y0,TH,X,RT,AR,NMS,NX,NSD,I,DWB,RHS,RSL)
      DO 2 I=1,NVP
      PRINT300,(DWI(I,J,1),J=1,NX)
    2 PRINT300,(DWB(I,J,1),J=1,NX)
      CALL FM(DWI,DWB,NX,NMS,VP,NVP,X,RT,UF,UM)
      PRINT550
  550 FORMAT('0',2X,'HULL RESPONSE TO UNIT STEP OF CIRCULATION')
      PRINT500,(X(I),UF(1,I),UM(1,I),I=1,NX)
      H=X(2)-X(1)
      CALL DERY(NX,H,UF,DUF,1)
      CALL DERY(NX,H,UM,DUM,1)
      PRINT600,(X(I),DUF(1,I),DUM(1,I),I=1,NX)
      CALL CONV(NX,H,DUF,UF,A,T1,T2,CUF,1)
      CALL CONV(NX,H,DUM,UM,A,T1,T2,CUM,1)
      DO 5 I=1,NX
      CUF(1,I)=CUF(1,I)*3.1416*C
    5 CUM(1,I)=CUM(1,I)*3.1416*C
      PRINT560
  560 FORMAT('0',2X,'HULL RESPONSE TO CIRCULATION OF SAIL TRANSIENT'
      PRINT500,(X(I),CUF(1,I),CUM(1,I),I=1,NX)
  100 FORMAT(4I5,6F5.3)
  200 FORMAT(16F5.3)
  300 FORMAT('0',2X,10E12.4)
  500 FORMAT('0',2X,'X=',E12.4,2X,'CY=',E12.4,2X,'CN=',E12.4)
  600 FORMAT('0',2X,'X=',F12.4,2X,'DCY/DS=',E12.4,2X,'DCN/DS=',E12.4
      STOP
      END
```

```
      SUBROUTINE UBND(XO,YO,T,X,RT,AR,NMS,NX,NSD,ICP,DWB,RHS,RSL)
      DIMENSION X(20),RT(20),DWB(40,20,3),R(6),E(6),F(6),W(3)
      Z1(A,B,C)=.5*(A*(C-B)+C+B)
      H=X(2)-X(1)
      ZO=(XO-1.)*SIN(T)
      XO1=(XO-1.)*COS(T)
      DO 1 I=1,NMS
      DO 1 N=1,NX
      IF(N.GT.NSD)GO TO 6
      XN=X(N)-1.
      RTF=RT(N)
      XH=X(N)
      RH=FRAD(XH)
      ARN=AR*(RTF-RH)/(RSL-RHS)
      YO1=YO-RSL+AR+RTF-ARN
      YH1=ARN-RTF+RH
      YHT=ARSIN(YH1/ARN)*.6366197
    6 CONTINUE
      IF(N.GT.NSD)X(N)=X(N-1)+H
      IF(N.GT.NSD)XN=XN+H
      IF(YO1.GE.YH1)GO TO 2
      HI=.1*(1.-YHT)
      Y1=YHT+HI
      VINT=FUBND(YHT,XO1,YO1,ZO,XN,ARN,YHT,I)+4.*FUBND(Y1,XO1,YO1,ZO,
     1ARN,YHT,I)+FUBND(1.,XO1,YO1,ZO,XN,ARN,YHT,I)
      DO 3 J=2,8,2
      Y1=YHT+J*HI
      Y2=Y1+HI
    3 VINT=VINT+2.*FUBND(Y1,XO1,YO1,ZO,XN,ARN,YHT,I)+4.*FUBND(Y2,XO1,
     1,ZO,XN,ARN,YHT,I)
      DWB(ICP,N,I)=HI*VINT/3.
      GO TO 1
    2 DIF=.6366197*ARSIN(YO1)-YHT
      UP=YHT+2.*DIF
      IF(UP.GT.1.)UP=2.*(DIF+YHT)-1.
      D1=(UP-YHT)*.5
      D2=(1.-UP)*.5
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      DO 4 II=1,6
      A1=R(II)
      E(II)=Z1(A1,YHT,UP)
    4 F(II)=Z1(A1,UP,1.)
      G1=0.
      G2=0.
      DO 5 II=1,3
```

```
      X1=F(2*II-1)
      X2=F(2*II)
      W1=F(2*II-1)
      W2=F(2*II)
      G1=G1+W(II)*(FUBND(X1,X01,Y01,Z0,XN,ARN,YHT,I)+FUBND(X2,X01,Y01,Z
     1,XN,ARN,YHT,I))
    5 G2=G2+W(II)*(FUBND(W1,X01,Y01,Z0,XN,ARN,YHT,I)+FUBND(W2,X01,Y01,Z
     1,XN,ARN,YHT,I))
      DWR(ICP,N,I)=(G1*D1+G2*D2)
    1 CONTINUE
      RETURN
      END
```

111<

IV [illegible] LEVEL 2.[illegible] FUND DATE = 74[illegible] 1[illegible]/42/[illegible]

```
      FUNCTION FUND(Y,XO,YO,ZO,XN,AP,YH,II)
      IF(II.EQ.2)GO TO 1
      IF(II.EQ.1)GS=SIN(1.5708*(Y-1.)/(YH-1.))
      IF(II.EQ.3)GS=SIN(4.7124*(Y-1.)/(YH-1.))
      GO TO 2
    1 IF(Y.GT.YH)GS=SIN(1.5708*Y/YH)
      IF(Y.LE.0.)GS=-SIN(Y*3.1416)
    2 CONTINUE
      ETA=AP*SIN(Y*1.5708)-YO
      DEN=ETA*ETA+ZO*ZO+(XN-XO)*(XN-XO)
      FUND=.125*AP*(XN-XO)*GS*COS(1.5708*Y)/DEN**1.5
      RETURN
      END
```

```
      SUBROUTINE UTRLD(XO,YO,T,X,AR,RT,NMS,NX,NSD,ICP,DWI,RHS,RSL)
      DIMENSION X(20),RT(20),DWI(40,20,3),R(6),E(6),F(6),W(3)
      Z1(A,B,C)=.5*(A*(C-B)+C+B)
      H=X(2)-X(1)
      ZO=(XO-1.)*SIN(T)
      XO1=(XO-1.)*COS(T)
      DO 1 I=1,NMS
      DO 1 N=2,NX
      IF(N.GT.NSD)GO TO 6
      XN=X(N)-1.
      XC=X(N)-.5*H
      XT=XN-H
      RTC=(RT(N)+RT(N-1))*.5
      RHC=HRAD(XC)
      ARN=AR*((RTC-RHC)/(RSL-RHS))
      YO1=YO-RSL+AR+RTC-ARN
      YH1=ARN-RTC+RHC
      YHT=ARSIN(YH1/ARN)*.6366197
    6 CONTINUE
      IF(N.GT.NSD)X(N)=X(N-1)+H
      IF(N.GT.NSD)XN=XN+H
      IF(N.GT.NSD)XT=XT+H
      IF(YO1.GE.YH1)GO TO 2
      HI=.1*(1.-YHT)
      Y1=YHT+HI
      VINT=FUTRLD(YHT,XO1,YO1,ZO,XN,XT,ARN,YHT,I)+4.*FUTRLD(Y1,XO1,YO1,Z
     1O,XN,XT,ARN,YHT,I)+FUTRLD(1.,XO1,YO1,ZO,XN,XT,ARN,YHT,I)
      DO 3 J=2,8,2
      Y1=YHT+J*HI
      Y2=Y1+HI
    3 VINT=VINT+2.*FUTRLD(Y1,XO1,YO1,ZO,XN,XT,ARN,YHT,I)+4.*FUTRLD(Y2,XO
     11,YO1,ZO,XN,XT,ARN,YHT,I)
      DWI(ICP,N,I)=HI*VINT/3.
      GO TO 1
    2 DIF=.6366197*ARSIN(YO1)-YHT
      UP=YHT+2.*DIF
      IF(UP.GT.1.)UP=2.*(DIF+YHT)-1.
      D1=(UP-YHT)*.5
      D2=(1.-UP)*.5
      R(1)=.2386192
      R(2)=-1.*R(1)
      R(3)=.6612094
      R(4)=-1.*R(3)
      R(5)=.9324695
      R(6)=-1.*R(5)
      W(1)=.4679139
      W(2)=.3607616
      W(3)=.1713245
      DO 4 II=1,6
      A1=R(II)
      E(II)=Z1(A1,YHT,UP)
    4 F(II)=Z1(A1,UP,1.)
      G1=0.
```

```
      G2=0.
      DO 5 II=1,3
      X1=E(2*II-1)
      X2=E(2*II)
      W1=F(2*II-1)
      W2=F(2*II)
      G1=G1+W(II)*(FUTRLD(X1,X01,Y01,Z0,XN,XT,ARN,YHT,I)+FUTRLD(X2,X01,Y
     101,Z0,XN,XT,ARN,YHT,I))
    5 G2=G2+W(II)*(FUTRLD(W1,X01,Y01,Z0,XN,XT,ARN,YHT,I)+FUTRLD(W2,X01,Y
     101,Z0,XN,XT,ARN,YHT,I))
      DWI(ICP,N,I)=(G1*D1+G2*D2)
    1 CONTINUE
      RETURN
      END
```

```
      FUNCTION FUTRLD(Y,XO,YO,ZO,XN,XT,AR,YH,II)
      IF(II.EQ.2)GO TO 1
      IF(II.EQ.1)GS=COS(1.5708*(Y-1.)/(YH-1.))/(YH-1.)
      IF(II.EQ.3)GS=3.*COS(4.7124*(Y-1.)/(YH-1.))/(YH-1.)
      GO TO 2
    1 IF(Y.GT.YH)GS=COS(1.5708*Y/YH)/YH
      IF(Y.GE.0.)GS=-2.*COS(Y*3.1416)
    2 CONTINUE
      ETA=AR*SIN(Y*1.5708)-YO
      DEN=ETA*ETA+ZO*ZO
      F1=(XN-XO)/SQRT((XN-XO)*(XN-XO)+DEN)
      F2=(XT-XO)/SQRT((XT-XO)*(XT-XO)+DEN)
      FUTRLD=-.125*GS*ETA*(F1-F2)/DEN
      RETURN
      END
```

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```
      SUBROUTINE FM(DWI,DWB,NSD,NMS,VP,NVP,X,RT,UF,UM)
      DIMENSION DWI(40,20,3),DWB(40,20,3),VP(40,2),X(20),RT(20),UF(3,20)
     1,UM(3,20)
      DIMENSION AF(20),AM(20)
      HI=VP(2,1)-VP(1,1)
      DO 1 I=1,NVP
      XA=VP(I,1)
      AF(I)=ASLP(XA)
    1 AM(I)=AF(I)*XA
      DO 3 K=1,NMS
C        FIND T=0+ RESPONSE
      SUMF=DWB(1,1,K)*AF(1)+4.*DWB(2,1,K)*AF(2)+DWB(NVP,1,K)*AF(NVP)
      SUMM=DWB(1,1,K)*AM(1)+4.*DWB(2,1,K)*AM(2)+DWB(NVP,1,K)*AM(NVP)
      NEND=NVP-2
      DO 2 I=3,NEND,2
      SUMF=SUMF+2.*DWB(I,1,K)*AF(I)+4.*DWB(I+1,1,K)*AF(I+1)
    2 SUMM=SUMM+2.*DWB(I,1,K)*AM(I)+4.*DWB(I+1,1,K)*AM(I+1)
      UF(K,1)=SUMF*HI/3.
      UM(K,1)=SUMM*HI/3.
C        FIND RESPONSE W/TRAILERS
      DO 3 I=2,NSD
      SUMF=SUMF+(DWB(1,I,K)-DWB(1,I-1,K)+DWI(1,I,K))*AF(1)
      SUMF=SUMF+4.*(DWB(2,I,K)-DWB(2,I-1,K)+DWI(2,I,K))*AF(2)
      SUMF=SUMF+(DWB(NVP,I,K)-DWB(NVP,I-1,K)+DWI(NVP,I,K))*AF(NVP)
      SUMM=SUMM+(DWB(1,I,K)-DWB(1,I-1,K)+DWI(1,I,K))*AM(1)
      SUMM=SUMM+4.*(DWB(2,I,K)-DWB(2,I-1,K)+DWI(2,I,K))*AM(2)
      SUMM=SUMM+(DWB(NVP,I,K)-DW (NVP,I-1,K)+DWI(NVP,I,K))*AM(NVP)
      DO 4 J=3,NEND,2
      SUMF=SUMF+2.*(DWB(J,I,K)-DWB(J,I-1,K)+DWI(J,I,K))*AF(J)
      SUMF=SUMF+4.*(DWB(J+1,I,K)-DWB(J+1,I,K)+DWI(J+1,I,K))*AF(J+1)
      SUMM=SUMM+2.*(DWB(J,I,K)-DWB(J,I-1,K)+DWI(J,I,K))*AM(J)
    4 SUMM=SUMM+4.*(DWB(J+1,I,K)-DWB(J+1,I,K)+DWI(J+1,I,K))*AM(J+1)
      UF(K,I)=SUMF*HI/3.
    3 UM(K,I)=SUMM*HI/3.
      RETURN
      END
```

```
      FUNCTION HRAD(X)
      BOW=-5.125
      STERN=7.0625
      X1=-4.25
      X2=.875
      X3=6.
      R1=.4351
      R2=.8438
      R3=.3281
      IF(X.LE.X1)GO TO 1
      IF(X.GE.X3)GO TO 2
      A=(R1+R3-2.*R2)*.5/(X2-X1)**2
      B=(R1-R3)/(X1-X3)
      C=R2
      XD=X-X2
      R=A*XD*XD+B*XD+C
      GO TO 3
    1 R=R1*SQRT((X-BOW)/(X1-BOW))
      GO TO 3
    2 R=R3*SQRT((X-STERN)/(X3-STERN))
    3 HRAD=R
      RETURN
      END
```

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```
      SUBROUTINE WCT(A,B,X,R,N)
      DIMENSION X(20),R(20)
      FINT(E,F,G,I,D)=E/((F-1.-(I-1)*D)*(F-1.-(I-1)*D)+G*G)**1.5
      H=.025*(B-A)
      HT=(G-1.)/N
      N1=N+1
      DO 1 J=1,N1
      RO=R(J)
      E1=ASLP(A)
      A1=A+H
      E2=ASLP(A1)
      E3=ASLP(B)
      XINT=FINT(E1,A,RO,J,HT)+4.*FINT(E2,A1,RO,J,HT)+FINT(E3,B,RO,J,HT)
      DO 2 IJ=2,38,2
      A1=A+IJ*H
      A2=A1+H
      E1=ASLP(A1)
      E2=ASLP(A2)
    2 XINT=XINT+2.*FINT(E1,A1,RO,J,HT)+4.*FINT(E2,A2,RO,J,HT)
      XINT=H*HT*XINT/18.85
      R(J+1)=R(J)*(1.+XINT)
    1 X(J+1)=X(J)+HT
      RETURN
      END
```

```
      FUNCTION ASLP(X)
      BOW=-5.125
      STERN=7.0625
      X1=-4.25
      X2=.875
      X3=6.
      R1=.4351
      R2=.8438
      R3=.3281
      IF(X.LE.X1)GO TO 1
      IF(X.GE.X3)GO TO 2
      A=(R1+R3-2.*R2)*.5/(X2-X1)**2
      B=(R1-R3)/(X1-X3)
      C=R2
      XD=X-X2
      ASLP=6.2832*(2.*A*A*XD*XD*XD+3.*A*B*XD*XD+(B*B+2.*A*C)*XD+C*B)
      GO TO 3
    1 ASLP=3.1416*R1*R1/(X1-BOW)
      GO TO 3
    2 ASLP=3.1416*R3*R3/(X3-STERN)
    3 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DERY(N,H,F,DF,II)
      DIMENSION F(3,40),DF(3,40)
      HI=1./H
      DF(II,N)=HI*(F(II,N)-F(II,N-1))
      DF(II,1)=HI*(F(II,2)-F(II,1))
      N1=N-1
      DO 1 I=2,N1
    1 DF(II,I)=.5*HI*(F(II,I+1)-F(II,I-1))
      RETURN
      END
```

```
      SUBROUTINE CONV(N,H,F,FI,A1,T1,T2,ANS,IJ)
      DIMENSION F(3,40),FI(3,40),ANS(3,40),G(40)
      G(1)=1.
      E1=EXP(T1*H)
      E2=EXP(T2*H)
      IE1=-12/(T1*H)
      IE2=-12/(T2*H)
      DO 1 I=2,N
    1 G(I)=0.
      DO 2 I=2,IE1
    2 G(I)=G(I)+A1*E1**(I-1)
      DO 3 I=2,IE2
      I1=I-1
    3 G(I)=G(I)+(1.-A1)*E2**(I-1)
      ANS(IJ,1)=0.
      DO 4 I=2,N
    4 ANS(IJ,I)=FI(IJ,I)-FI(IJ,1)*G(I)
C        DO FIRST INTEGRABLE STEP
      ANS(IJ,2)=ANS(IJ,2)-.5*H*(F(IJ,1)*G(2)+F(IJ,2)*G(1))
C        DO CASE OF EVEN NUMBER OF BASE POINTS
      DO 5 I=4,N,2
      ANS(IJ,I)=ANS(IJ,I)-.5*H*(F(IJ,1)*G(I)+F(IJ,2)*G(I-1))
      NP=I
      AN1=F(IJ,2)*G(I-1)+4.*F(IJ,3)*G(I-2)+F(IJ,I)*G(1)
      IF(NP.EQ.4)GO TO 5
      NE=NP-2
      DO 6 J=4,NE,2
    6 AN1=AN1+2.*F(IJ,J)*G(I-J+1)+4.*F(IJ,J+1)*G(I-J)
    5 ANS(IJ,I)=ANS(IJ,I)-H*AN1/3.
C        DO CASE OF ODD NUMBER OF BASE POINTS
      DO 8 I=3,N,2
      AN1=F(IJ,1)*G(I)+4.*F(IJ,2)*G(I-1)+F(IJ,I)*G(1)
      IF(I.EQ.3)GO TO 8
      NE=I-2
      DO 9 J=3,NE,2
    9 AN1=AN1+2.*F(IJ,J)*G(I-J+1)+4.*F(IJ,J+1)*G(I-J)
    8 ANS(IJ,I)=ANS(IJ,I)-H*AN1/3.
  200 FORMAT('0',2X,'G(',I2,')=',F12.4)
      RETURN
      END
```